Final Exam

CAS705

November 28, 2003

Examiner: Michael Soltys

Duration: 3hrs

The exam consists of one page and one question.

Question: Explain the relationships among the complexity classes in the box below.

$$\mathbf{L} \overset{(1)}{\subseteq} \mathbf{NL} \overset{(2)}{\subseteq} \mathbf{P} \overset{(3)}{\subseteq} \mathbf{NP} \overset{(4)}{\subseteq} \mathbf{PSPACE} \overset{(5)}{=} \mathbf{NPSPACE} \overset{(6)}{=} \textbf{co-NPSPACE} \overset{(7)}{\subseteq} \mathbf{EXPTIME}$$

$$\mathbf{EXPTIME} \overset{(8)}{\subseteq} \mathbf{EXPSPACE}$$

$$\textbf{Regular} \overset{(9)}{\subsetneqq} \mathbf{CFL} \overset{(10)}{\subsetneqq} \mathbf{P} \overset{(11)}{\subsetneqq} \textbf{Decidable} \overset{(12)}{\subsetneqq} \mathbf{RE} \overset{(13)}{\subsetneqq} \mathcal{P}(\Sigma^*)$$

$$\mathbf{NL} \overset{(14)}{\subsetneqq} \mathbf{PSPACE} \quad \text{and} \quad \mathbf{P} \overset{(15)}{\subsetneqq} \mathbf{EXPTIME}$$

Instructions:

1. Make sure that you define all the complexity classes, and that you provide complete proofs of the inclusions.
2. The relationships among the complexity classes are numbered to facilitate the organization of your answer.
3. If an inclusion is proper, provide a proof of the separation (thus, you must provide proofs of separations (9–15), as well as the corresponding inclusions).
4. Note that the "mathematical style" of your answer will be an important component of your final grade.
5. Make good use of things that you have already proven. For example, to show the inclusion in (14), you can use (2,3,4) if you have already shown it. Then you can complete (14) by showing that the inclusion is in fact proper.

End of Exam