Name _____ Student No. _____

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks 10.

1. $L_{\text{pal}} = \{x \in \{0,1\}^* | x = x^R\}$ requires $\Omega(n^2)$ steps on a single tape Turing machine and can be decided in $O(n)$ steps on a two-tape Turing machine. Explain what these two statements mean, and justify the *second*.

2. In this question we are going to prove the *first* statement from question 1.

   (a) Define the *i-th crossing sequence of M on x*: $\{(q_1, \sigma_1), (q_2, \sigma_2), \ldots, (q_m, \sigma_m)\}$. Show that $\sigma_{2i} = \sigma_{2i-1}$.

   (b) Now consider inputs of the form $x0^n x^R$, where $|x| = n$. Let $T_M(x)$ be the number of steps that M takes to decide (and accept) $x0^n x^R$. Show that there must be some i, $n < i \leq 2n$, i.e., some square in 0^n, for which the i-th crossing sequence (call it S) has length $m \leq \frac{T_M(x)}{n}$.

   (c) Show that M, n, i, S describe x uniquely.

   (d) Show that M, n, i, S can be encoded with $c_M + 2\log n + c \cdot \frac{T_M(x)}{n}$ bits, where c_M is a constant that depends only on M.

   (e) Show that for every encoding, and any n, there exists a string x of length n whose encoding requires at least n bits.

   (f) Conclude the first statement of question 1.