Assignment 2 **Due: Oct 23** **CAS705 Fall 2009**

1. Consider the problem NAE3Sat defined on page 122 of the text book. Lemma 8.12 shows that NAE3Sat is NP-complete.

   Use this fact to show that the language MaxCut, which we define below, is also NP-complete.

   A *cut* in an undirected graph is a separation of the vertices V into two disjoing subsets S and T. The size of a cut is the number of edges that have one endpoint in S and the other in T. We let

   $$\textsc{MaxCut} = \{\langle G, k\rangle : \ G \text{ has a cut of size } k \text{ or more } \}.$$

2. The problem 2Sat is the problem of satisfiability of Boolean formulas in CNF with exactly two literals per clause. On page 90 of the book we show (Lemma 6.15) that 2Sat is in P, using resolution.

   A literal l is a variable x or its negation $\bar{x}$. If $l = \bar{x}$, then $\bar{l} = x$.

   In this problem you are going to show that 2Sat is in P using a different method. Given a 2CNF formula α associate with it a directed graph $G_\alpha = (V, E)$, where V is the set of all literals l such that either l or $\bar{l}$ occurs in α, and (l_1, l_2) is a directed edge in G iff the clause $(\bar{l_1} \vee l_2)$ (or $(l_2 \vee \bar{l_1})$) occurs in α.

   (a) Show that given any literals l_1 and l_2, if there is a directed path from l_1 to l_2 in G_α then there is a directed path from $\bar{l_2}$ to $\bar{l_1}$, and every truth assignment to α which satisfies α as well as l_1 also satisfies l_2.

   (b) Show that α is unsatisfiable iff G_α has a directed cycle which includes both x and $\bar{x}$, for some variable x.

   (c) Conclude with a polytime algorithm for 2Sat.