

1. This exercise is related to the Immerman-Szelepcsényi theorem.

Suppose that the membership in some language  $L$  can be determined by a nondeterministic TM  $M$  with time complexity  $t_M(n)$  and space complexity  $s_M(n)$ .

Furthermore, suppose that the number of strings in  $L$  of a given length is given by  $f : \mathbb{N} \rightarrow \mathbb{N}$ , that requires time  $t_f(n)$  and space  $s_f(n)$ .

Under these assumptions give a nondeterministic TM for  $\overline{L}$ , and determine upper bounds for the time and space complexity for such a TM.

2. Suppose that the language  $L$  in the previous question is context-sensitive. Let  $T_{n,i} := \{x : |x| \leq n, S \stackrel{i}{\Rightarrow} x\}$ , so that for some  $m$  we will have  $T_{n,m} = T_{n,m+1}$ . Let  $g(n) := |T_{n,m}|$  for this  $m$ .

Show that  $g$  can be computed in nondeterministic linear space.

Conclude that if  $L$  is context-sensitive, so is  $\overline{L}$ .

3. Do exercises 3.14, 3.15, 3.16 in the textbook. These are exercises related to interactive proof systems.