Problem Set 3 **Due: March 30** **cs2me3 Winter 2004**

Instructions

1. You are encouraged to work in groups of three. If you cannot find two partners, you can work in a group of two. As a last resort, you can work alone.
2. The names of all the partners must be indicated on the assignment.
3. For **Part A** of the assignment, you must submit an electronic copy of your C program via WebCT, and a hard copy of your program should be included when you hand in the entire assignment (Parts A and B) in class.

Part A

1. Write a **C** program which takes as input a string α, and checks that α is a well-formed propositional formula. If it is, it checks whether α is satisfiable, and if it is, it outputs one satisfying assignment.

   We are going to use `&` for $\wedge$, `|` for $\vee$, and `-` for $\neg$. The legal variables will be `a-z` (lower-case letters). Thus the grammar for the well-formed inputs is:

   $S \longrightarrow$ `a-z` $|$ `(` S `&` S `)` $|$ `(` S `|` S `)` $|$ `-`S

   So,

   ```
   myprogram "(&&(Wp"
   ```

   should output the message: `not a well-formed formula`, and

   ```
   myprogram "(a&-a)"
   ```

   should output the message: `not satisfiable`, and finally

   ```
   myprogram "(w&-(c|d))"
   ```

   should give a satisfying assignment as output, for example:

   ```
   w=1
   c=0
   d=0
   ```

Part B

1. Write out in full detail the proof of correctness of the multiplication program `mult(A,B)`. Your proof should consist of a table with three columns: the first column should contain the line number, the middle column should contain a line of the proof, and the third column should contain the justification.

   Here is an example segment of the proof:

1	$\{y + a(b-1) = AB \wedge (b-1) \geq 0\}$`b=b-1;`$\{y + ab = AB \wedge b \geq 0\}$	assignment
2	$\{(y+a) + a(b-1) = AB \wedge (b-1) \geq 0\}$`y=y+a;`$\{y + a(b-1) = AB \wedge (b-1) \geq 0\}$	assignment
3	$(y + ab = AB \wedge b - 1 \geq 0) \supset ((y+a) + a(b-1) = AB \wedge b - 1 \geq 0)$	theorem
4	$\{y + ab = AB \wedge b - 1 \geq 0\}$`y=y+a;`$\{y + a(b-1) = AB \wedge b - 1 \geq 0\}$	consequence left 2 and 3
5	$\{y + ab = AB \wedge b - 1 \geq 0\}$`y=y+a;b=b-1;`$\{y + ab = AB \wedge b \geq 0\}$	composition on 4 and 1

2. Suppose that P is a unary predicate symbol, s is a unary function, and $<$ is a binary predicate symbol (used in infix notation). Define the formulas A, B, C by:

$$A = \forall x(x < sx)$$
$$B = \forall x(Px \vee Psx)$$
$$C = \forall x \forall y \forall z(\neg(x < y \wedge y < z) \vee x < z)$$

   Give an LK proof of the sequent:

$$A, B, C \rightarrow \forall x \exists y(x < y \wedge Py)$$

   You may shorten your presentation by using the letters A, B, C where appropriate, and introducing other names for formulas.

   Hint. Start by convincing yourself that the sequent is valid. Once you see that, you will see a strategy for a proof.