Problem Set 3 **SOLUTIONS** **cs2me3 Winter 2004**

Part B, Q1

1	$\{y + a(b-1) = AB \wedge (b-1) \geq 0\}$`b=b-1;`$\{y + ab = AB \wedge b \geq 0\}$	assignment
2	$\{(y+a) + a(b-1) = AB \wedge (b-1) \geq 0\}$`y=y+a;`$\{y + a(b-1) = AB \wedge (b-1) \geq 0\}$	assignment
3	$(y + ab = AB \wedge b - 1 \geq 0) \supset ((y+a) + a(b-1) = AB \wedge b - 1 \geq 0)$	theorem
4	$\{y + ab = AB \wedge b - 1 \geq 0\}$`y=y+a;`$\{y + a(b-1) = AB \wedge b - 1 \geq 0\}$	consequence left 2 and 3
5	$\{y + ab = AB \wedge b - 1 \geq 0\}$`y=y+a;b=b-1;`$\{y + ab = AB \wedge b \geq 0\}$	composition on 4 and 1
6	$(y + ab = AB) \wedge b \geq 0 \wedge b > 0 \supset (y + ab = AB) \wedge b - 1 \geq 0$	theorem
7	$\{(y + ab = AB) \wedge b \geq 0 \wedge b > 0\}$`y=y+a; b=b-1;`$\{y + ab = AB \wedge b \geq 0\}$	consequence left 5 and 6
8	$\{(y + ab = AB) \wedge b \geq 0\}$ `while (b>0) {` `y=y+a;` `b=b-1; }` $\{y + ab = AB \wedge b \geq 0 \wedge \neg(b > 0)\}$	while on 7
9	$\{(0 + ab = AB) \wedge b \geq 0\}$ `int y=0;` $\{(y + ab = AB) \wedge b \geq 0\}$	assignment

10	$\{(0 + ab = AB) \wedge b \geq 0\}$ `int y=0;`
	`while (b>0) {`
	`y=y+a;`
	`b=b-1; }` $\{y + ab = AB \wedge b \geq 0 \wedge \neg(b > 0)\}$
		composition on 9 and 8

11	$\{(0 + aB = AB) \wedge B \geq 0\}$ `int b=B;` $\{(0 + ab = AB) \wedge b \geq 0\}$	assignment

12	$\{(0 + aB = AB) \wedge B \geq 0\}$ `int b=B;`
	`int y=0;`
	`while (b>0) {`
	`y=y+a;`
	`b=b-1; }` $\{y + ab = AB \wedge b \geq 0 \wedge \neg(b > 0)\}$
		composition on 11 and 10

13	$\{(0 + AB = AB) \wedge B \geq 0\}$ `int a=A;` $\{(0 + aB = AB) \wedge B \geq 0\}$	assignment
14	$\{(0 + AB = AB) \wedge B \geq 0\}$ `mult(int A, int B)` $\{y + ab = AB \wedge b \geq 0 \wedge \neg(b > 0)\}$	composition on 13 and 12
15	$B \geq 0 \supset ((0 + AB = AB) \wedge B \geq 0)$	theorem
16	$(y + ab = AB \wedge b \geq 0 \wedge \neg(b > 0)) \supset y = AB$	theorem
17	$\{B \geq 0\}$ `mult(int A, int B)` $\{y + ab = AB \wedge b \geq 0 \wedge \neg(b > 0)\}$	consequence left on 15 and 14
18	$\{B \geq 0\}$ `mult(int A, int B)` $\{y = AB\}$	consequence right on 16 and 17

Partb B, Q2

We present an outline of the derivation. Note that to have full marks, the entire derivation, with all the details is needed. It is difficult to typeset such a derivation, but easy to present it "by hand."

First derive $A \to a < sa$ and $A \to sa < ssa$.

From the above two sequents derive $A, C \to (a < sa \wedge a < ssa)$.

Finally, derive $B \to Psa \vee Pssa$.

Putting it all together, we obtain:

$$A, B, C \to (a < sa \wedge a < ssa) \wedge (Psa \vee Pssa).$$

On the other hand, it is easy to show that

$$(a < sa \wedge a < ssa) \wedge (Psa \vee Pssa) \to a < sa \wedge Psa, a < ssa \wedge Pssa$$

by getting rid of the connectives using connective-introduction rules in reverse. Now, using a cut we obtain:

$$A, B, C \to a < sa \wedge Psa, a < ssa \wedge Pssa$$

Use $\exists$-intro right twice on the above sequent, first with $t = sa$, then with $t = ssa$ to obtain:

$$A, B, C \to \exists y(a < y \wedge Py), \exists y(a < y \wedge Py)$$

and with a contraction:

$$A, B, C \to \exists y(a < y \wedge Py)$$

As A, B, C are sentences, we can use $\forall$-right intro on a, to obtain the desired conclusion:

$$A, B, C \to \forall x \exists y(x < y \wedge Py)$$