Review Exercises II **cs2me3** **Winter 2004**

Not to hand in. These are review exercises for the exam.

1. For each of the following programs, give a precondition, a postcondition, and a loop invariant. Show by induction that your loop invariant holds. Then show that the postcondition follows from the precondition and the loop invariant.

   (a)
   ```
   int power(int x) {
         int current = 1;
         int count = 0;
         while(count < x) {
            current = current*2;
            count += 1; }}
   ```
   (b)
   ```
   int multiply(int m, int n) {
         int x = m;
         int y = n;
         int z = 0;
         while(x != 0) {
            if(x % 2 = 1) {
                z = z+y; }
            x = x/2;
            y = y*2; }}
   ```
   (c)
   ```
   int square(int n) {
         int x = 0;
         int k = 2;
         while (k <= 2*n) {
            x = x+k-1;
            k += 2; }}
   ```

2. Let $\mathcal{L} = [;R,=]$, a first order language with a binary predicate symbol R and the equality relation (and no function symbols). Think of R as an edge relation (i.e., Rxy is true iff (x, y) is an edge).

   Give an $\mathcal{L}$-sentence α such that if $\mathcal{M}$ is a finite structure, then $\mathcal{M} \vDash \alpha$ iff $\mathcal{M}$ consisting of a disjoint union of directed cycles.

3. Prove $(u(t/x))^{\mathcal{M}}[\sigma] = u^{\mathcal{M}}[\sigma(m/x)]$ where $m = t^{\mathcal{M}}[\sigma]$, by structural induction on u.

4. Prove the Substitution Theorem. (Hint. Use structural induction on α and the BSDs.)

5. Verify the Soundness Principle for the four rules for introducing quantifiers in LK.

6. Give a specific example of a sequent $\Gamma \to \Delta, \alpha(b)$ which is valid, but the bottom sequent $\Gamma \to \Delta, \forall x\alpha(x)$ is not valid, because the restriction on b is violated (b occurs in Γ or Δ or $\forall x\alpha(x)$). Do the same for $\exists$ left.

7. Give LK proofs for the following valid sequents:

   (a) $\forall x Px \wedge \forall x Qx \to \forall x(Px \wedge Qx)$
   (b) $\forall x(Px \wedge Qx) \to \forall x Px \wedge \forall x Qx$
   (c) $\exists x Px \vee \exists x Qx \to \exists x(Px \vee Qx)$
   (d) $\exists x(Px \vee Qx) \to \exists x Px \vee \exists x Qx$
   (e) $\exists x(Px \wedge Qx) \to \exists x Px \wedge \exists x Qx$
   (f) $\exists y \forall x Pxy \to \forall x \exists y Pxy$
   (g) $\forall x Px \to \exists x Px$