$\mathcal{L}$-terms:

1. Every variable is a term:
   $x, y, z, \ldots, a, b, c, \ldots$.

2. If f is an n-ary function symbol and $t_1, t_2, \ldots, t_n$ are terms, then so is $ft_1t_2\ldots t_n$.

A 0-ary function symbol is a constant (use e as a metasymbol for constants).

Also 0 and 1 are constants (in $\mathcal{L}_A$).

Eg. f is binary and g is unary, then $fgex, fxy, gfege$ are terms.

Ex. Show the Unique Readability Theorem for terms.

Let $r, s, t, \ldots$ denote terms.

$(t_1 * t_2) =_{\text{synt}} *t_1t_2$
$(t_1 + t_2) =_{\text{synt}} +t_1t_2$
Thus, we use **infix** notation instead of the formal **prefix** notation for $\mathcal{L}_A$ function symbols $*, +$.

Recall $\mathcal{L}_A = [0, s, +, *; =]$.

Eg. $\mathcal{L}_A$-terms: $sss0, ((x + sy) * (ssz + s0))$

$\mathcal{L}$-formulas:

1. $Pt_1t_2\ldots t_n$ is an **atomic** formula, P is an n-ary predicate symbol, $t_1, t_2, \ldots, t_n$ are terms.

2. If α, β are formulas, then so are $\neg\alpha, (\alpha \vee \beta), (\alpha \wedge \beta)$.

3. If α is a formula, and x a variable, then $\forall x\alpha$ and $\exists x\alpha$ are also formulas.

Eg. $(\neg\forall xPx \vee \exists x\neg Px)$, $(\forall x\neg Qxy \wedge \neg\forall zQfyz)$.

Ex. Show that the set of (well-formed) $\mathcal{L}$-formulas can be given by an $G_{\mathcal{L}}$ grammar of type 2.

Let $r = s$ stand for $= rs$ and $r \neq s$ for $\neg = rs$.

An occurrence of x in α is **bound** if it is in a subformula of α of the form $\forall x \beta$ of $\exists x \beta$ (i.e., in the **scope** of a quantifier).

Otherwise, the occurrence is **free**.

Eg. $\exists y(x = y + y)$: x is free, but y is bound.

In $Px \wedge \forall x Qx$ the variable x occurs as free and bound.

A term t or formula α are **close** if they contain no free variables. A closed formula is called a **sentence**.