Problem Set 3 **Due: March 27** **cs2me3 Winter 2006**

Instructions

1. You are encouraged to work in groups of two. If you cannot find a partner, you can work alone.
2. Please submit one copy of the assignment; if you are working with a partner, both names should appear on the assignment.
3. For **Part A** of the assignment, you must submit an electronic copy of your Java application via WebCT (by the time of the lecture on the due date of the assignment).

Part A

Write a Java application which implements the dynamic programming algorithm (presented in class) for computing the determinant via clow sequences.

Your program should work as follows: given an ASCII text file as input, `matrix.txt`, which contains an *integer* matrix, it should output (to the standard output) the value of the determinant of that matrix. The file `matrix.txt` must contain the size of the matrix, and its entries row by row. For example, the matrix

$$\begin{pmatrix} -1 & 3 & 27 \\ 14 & 0 & 1 \\ 5 & 7 & -2 \end{pmatrix}$$

should be presented in `matrix.txt` as follows:

```
3
-1 3 27
14 0 1
5 7 -2
```

(Note that the entries should be separated by a single space.)

Part B

1. Do the following exercises from the notes: 3.8.6, 3.8.7, 3.8.8, 3.8.9, 3.8.10, 3.8.11, 3.8.12, 3.8.13.

   Solutions: 3.8.8 The only two clows that change are C_i and C_j. C_j is inserted into C_i, resulting in C_i' which has all the edges of C_i and C_j.

   3.8.9 Replace $(8, 11, 10, 12, 9, 10, 14)$ by $(8, 11, 10, 14)$ and $(9, 10, 12)$.

3.8.10 It is a clow sequence because the only change was in C_i, where a cycle C was detached and made into a new clow with its head being its least node. The only possible problem was that the head of the new clow, head(C), is the same as the head of some clow comming after C_i. But that would throw is into case 1. The number of edges is clearly the same, and the parity is opposite because we added one more clow, C.

3.8.11 It would mean that we have case 2 (case 1 would not be possible, since there are now clows after k).

3.8.12 By the definition of the involution φ, we would focus on the cycle in C_i' (i being the least index such that $C_{i+1}, \ldots, C_k$ are all disjoint simple cycles), and we would detach the cycle C and obtain a new clow, which would be precisely C_j.

3.8.13 We already know that $\det(A) = \sum_C \operatorname{sign}(C) w(C)$, where C is a clow sequence. We now have to show that we can in fact restrict ourselves to clow sequences C such that head(C_1) = 1, i.e., the head of the first clow is the first vertex. This follows directly from the fact that clow sequences where that is not the case cannot be cycle covers (by the pigeonhole principle), and so they are eliminated by the existence of the involution.