Name ______________________ Student No. ______________________

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: **60**

[20] 1. State the principle of induction (PI), and the principle of complete induction (PCI). Show that PI implies PCI.

IP is stated on page 1 in the textbook, and PCI on page 2. To show that PI implies PCI: suppose that we have PI; assume that $P(0)$ and $\forall n((\forall i \leq n)P(i) \rightarrow P(n+1))$. We want to show that $\forall n P(n)$, so we prove this with the PI. $P(0)$ is given. To show $\forall j(P(j) \rightarrow P(j+1))$ suppose that it does not hold; then there exists a j such that $P(j)$ and $\neg P(j)$; let j be the smallest such j; one exists by the LNP, and $j \neq 0$ by what is given. So $P(0), P(1), P(2), \ldots, P(j)$ but $\neg P(j+1)$. But this contradicts $\forall n((\forall i \leq n)P(i) \rightarrow P(n+1))$, and so it is not possible. Hence $\forall j(P(j) \rightarrow P(j+1))$ and so by the PI we have $\forall n P(n)$ and hence we have the PCI.

[20] 2. At a country club, each member dislikes at most three other members. There are two tennis courts; show that each member can be assigned to one of the two courts in such a way that at most one person they dislike is also playing on the same court.

See solution to probem 1.12 on page 20.

[20] 3. Show the correctness of Euclid's algorithm, i.e., partial correctness and termination.

Algorithm 1 Euclid

Pre-condition: $a > 0 \wedge b > 0$

```
m ← a ; n ← b ; r ← rem(m, n)
while (r > 0) do
    m ← n ; n ← r ; r ← rem(m, n)
end while
return n
```

Post-condition: $n = \gcd(a, b)$

This is section 1.3.2 on page 7.

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