

Name _____ Student No. _____

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: **60**

- [20] 1. Use induction to prove that for $n \geq 1$,

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = (1 + 2 + 3 + \cdots + n)^2.$$

See solution to problem 1.2 on page 17.

- [20] 2. Handshakes are exchanged at a meeting. We call a person an *odd person* if he has exchanged an odd number of handshakes. Show that, at any moment, there is an even number of odd persons.

See solution to problem 1.14 on page 21.

- [20] 3. Show the correctness of the division algorithm, i.e., partial correctness and termination.

Algorithm 1 Division

Pre-condition: $x \geq 0 \wedge y > 0$

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1:  $q \leftarrow 0$ 
2:  $r \leftarrow x$ 
3: while  $y \leq r$  do
4:    $r \leftarrow r - y$ 
5:    $q \leftarrow q + 1$ 
6: end while
7: return  $q, r$ 
```

Post-condition: $x = (q \cdot y) + r \wedge 0 \leq r < y$

This is section 1.3.1 on page 6.

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