Name ________________________ Student No. ________________________

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: **60**

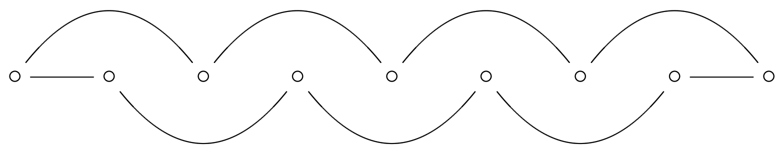
[20] 1. Consider the following instance of the stable matching problem, where the boys are $\{a, b, c\}$ and the girls are $\{x, y, z\}$ and the lists of preferences are given below.

$$\begin{array}{ll} a\colon\ y < x < z & x\colon\ b < a < c \\ b\colon\ z < y < x & y\colon\ c < b < a \\ c\colon\ x < z < y & z\colon\ a < c < b \end{array}$$

Is there a stable solution where (a, x) is a pair?

Yes, $\{(a, x), (b, y), (c, z)\}$.

2. How many minimum cost spanning trees does the following network have? Assume that the cost of each edge is the inverse of the throughput rate, and that it is the ***same for all edges***, $\frac{1}{5.006\text{Mbits}}$.



Just remove any one of the edges, and you obtain a minimum cost spanning tree. As 9 edges can be removed, there are 9 different minimum cost spanning trees.

3. First state Kruskal's algorithm. Then show that if $G = (V, E)$ is connected, then the assertion

   > "after the first i iterations of the for-loop the graph $(V, T \cup \{e_{i+1}, e_{i+2}, \ldots, e_m\})$ is connected"

   is a loop invariant.

   Problem 2.6 in the book, presented in class.