Name ______________________ Student No. ______________________

No aids allowed. Answer all questions on test paper. Use backs of sheets if necessary.

Total Marks: **60**

[20] 1. Explain the concept of a "promising partial solution," illustrating it with two examples: the greedy algorithm for minimum cost spanning trees and the greedy algorithm for jobs with deadlines and profits.

Solution: In both cases it means that the partial solution obtained thus far can be extended to an optimal solution with items not considered yet; these are edges in Kruskal and jobs in the other.

[20] 2. Consider the following input to the "job with deadlines and profits" problem:

$$\{\underbrace{(1,10)}_{1},\underbrace{(1,10)}_{2},\underbrace{(2,8)}_{3},\underbrace{(2,8)}_{4},\underbrace{(4,6)}_{5},\underbrace{(4,6)}_{6},\underbrace{(4,6)}_{7},\underbrace{(4,6)}_{8}\}$$

where the jobs have been numbered underneath for convenience. Your job is to trace the greedy algorithm we have seen in class on this input. On the left place the job numbers in the appropriate slots; on the right, show how the optimal solution is adjusted to keep the "promising" property.

$S^0 = \boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{0}$ $S^0_{\text{opt}} = \boxed{2}\boxed{4}\boxed{5}\boxed{8}\boxed{0}\boxed{0}\boxed{0}\boxed{0}$

$S^1 = \boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}$ $S^1_{\text{opt}} = \boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}$

$S^2 = \boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}$ $S^2_{\text{opt}} = \boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}$

$S^3 = \boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}$ $S^3_{\text{opt}} = \boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}$

$S^4 = \boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}$ $S^4_{\text{opt}} = \boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}$

Solution:

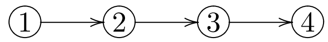
$S^1 = \boxed{1}\boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{0}$ $S^1_{\text{opt}} = \boxed{1}\boxed{4}\boxed{5}\boxed{8}\boxed{0}\boxed{0}\boxed{0}\boxed{0}$

$S^2 = \boxed{1}\boxed{3}\boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{0}$ $S^2_{\text{opt}} = \boxed{1}\boxed{3}\boxed{5}\boxed{8}\boxed{0}\boxed{0}\boxed{0}\boxed{0}$

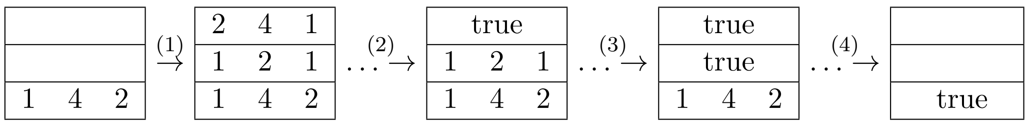
$S^3 = \boxed{1}\boxed{3}\boxed{0}\boxed{5}\boxed{0}\boxed{0}\boxed{0}\boxed{0}$ $S^3_{\text{opt}} = \boxed{1}\boxed{3}\boxed{8}\boxed{5}\boxed{0}\boxed{0}\boxed{0}\boxed{0}$

$S^4 = \boxed{1}\boxed{3}\boxed{6}\boxed{5}\boxed{0}\boxed{0}\boxed{0}\boxed{0}$ $S^4_{\text{opt}} = \boxed{1}\boxed{3}\boxed{6}\boxed{5}\boxed{0}\boxed{0}\boxed{0}\boxed{0}$

[20] 3. Suppose that we want to compute reachability from node 1 to node 4 in the following directed graph:



The trace of the stack resulting from Savitch's reachability procedure is given below:



Explain, briefly but to the point, the contents of the stack and the transitions, $1, 2, 3, 4$. In particular, what are the numbers in the stack?

Solution: The first stack contains the initial problem: it examines whether we can get from 1 to 4 with at most $2^2 = 4$ edges. In the first transition we push onto the stack the "break-down" of the original question: can we get from 1 to 2 with at most $2^1 = 2$ edges and from 2 to 4 with at most $2^1 = 2$ edges? The answers to both questions come back as "true"; we use dots to emphasize that several steps took place. Thus the final answer is true.