Name: ______________________________

Student Number: ______________________

COMP SCI 2MJ3 (Theory of Computation) Michael Soltys

DAY CLASS

DURATION OF EXAMINATION: 2 Hours

MCMASTER UNIVERSITY FINAL EXAMINATION December 2010

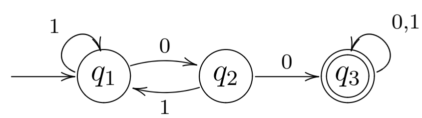
THIS EXAMINATION PAPER CONSISTS OF 2 PAGES AND 6 QUESTIONS.

YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

No aids allowed.

All questions worth 10 marks, for a total of 60.

1. Consider the following DFA:



   (a) What is the language accepted by this DFA?

   (b) Convert this DFA into a Regular Expression.

2. Consider the language $L_{\text{prime}} = \{1^p : p \text{ is a prime number}\}$, over the unary alphabet $\Sigma = \{1\}$. That is, $L_{\text{prime}} = \{11, 111, 11111, 1111111, \ldots\}$, i.e., L_{prime} contains all the prime numbers represented in unary. A number is *prime* if its only divisors are 1 and itself.

   (a) State the *Pumping Lemma* for regular languages.

   (b) State the *Myhill-Nerode* theorem—make sure that you define the equivalence relation "$\equiv_L$" (i.e., the *indistinguishability relation* for a given language L).

   (c) Show that L_{prime} is not regular, using the Pumping Lemma, or the Myhill-Nerode theorem (pick whichever you prefer).

3. Show that if L_1 is a context-free language, and L_2 is a regular language, then $L_1 \cap L_2$ is also a context-free language. Also show that if L_1 and L_2 are *both* context-free languages, then $L_1 \cap L_2$ is *not necessarily* a context-free language.

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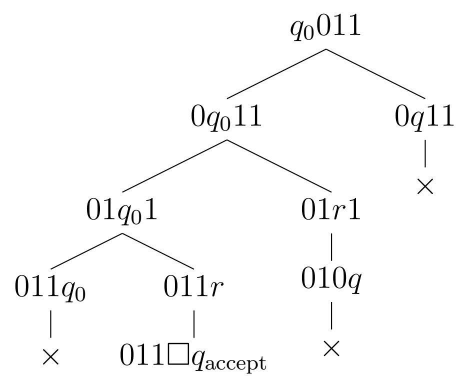
4. Explain what is the *Normal Form* for context-free grammars (CFGs).

   (a) Show that if G is a CFG in Normal Form, then for any string $w \in L(G)$ of length $n \geq 1$, exactly $2n-1$ steps are required in any derivation of w.

   (b) Conclude that the language $\{\langle G, w\rangle : G \text{ is a CFG and } S \overset{*}{\Rightarrow} w\}$ is *decidable*.

5. Consider the following nondeterministic Turing machine N presented in class. The transition function of N is given below, together with the "computation tree" on input 011.

$$\begin{aligned}
\delta(q_0, 0) &= \{(q_0, 0, \rightarrow), (q, 0, \rightarrow)\} \\
\delta(q_0, 1) &= \{(q_0, 1, \rightarrow), (r, 1, \rightarrow)\} \\
\delta(r, \Box) &= \{(q_{\text{accept}}, \Box, \rightarrow)\} \\
\delta(r, 0/1) &= \{(q, 0, \rightarrow)\}
\end{aligned}$$



   (a) What is the language of N? Justify your answer.

   (b) Present the computation tree for 010.

   (c) Explain, at a high level, how would a deterministic machine M *simulate* the machine N. You can refer to the example of the "computation tree" for 011 in your explanation.

6. Show that a language is decidable iff some enumerator enumerates the language in lexicographic order.