Name ______________________ Student No. ______________________

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: 80

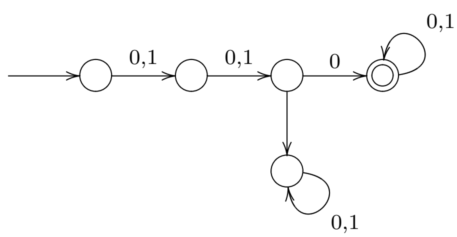
[20] 1. Give state diagrams of DFAs recognizing the following languages (assume $\Sigma = \{0, 1\}$):

(a) $\{w : w$ has length at least 3 and its third symbol is $0\}$

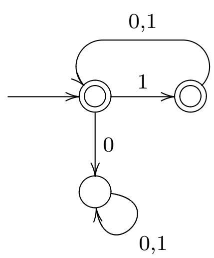
(b) $\{w :$ every odd position of w is a $1\}$

Solutions

(a)



(b)



[20] 2. Prove that every NFA can be converted to an equivalent one that has a single accept state.

Solution: This is problem 1.11 in Sipser: create a new (unique) accepting state q_{new} and connect the old accepting states, which are now plain states, by ε-arrows, to q_{new}.

[20] 3. For any strings $w \in \{0,1\}^*$, let w^R be the *reverse* of w; that is, if $w = a_1a_2\ldots a_n$ then $w^R = a_n \ldots a_2a_1$. For any language L, let $L^R = \{w^R : w \in L\}$; that is, L^R has the strings of L in reverse.

Show that if L is regular then so is L^R.

Solution: This is problem 1.31 in Sipser.

We do it as follows: since L is regular it has a DFA D; we take D and we modify it as follows to obtain a new NFA D': let the (only) accepting state of D' be the initial state of D. Let the initial state of D' be a new state, which we connect via ε-arrows to the accepting states of D (which are now only plain states in D'). Reverse all the arrows in D. Clearly D' accepts L^R; hence L^R is a regular language.

[20] 4. Consider $L^c = \Sigma^* - L$; that is, L^c, called the *complement* of L, is the set of all the strings in Σ^* except those in L.

Show that if L is regular then so is L^c.

Solution: This is problem 1.14(a) in Sipser: just swap the accepting and non-accepting states in the DFA that recognizes L to obtain a new DFA, D', that recognizes L^c; thus L^c is also regular.