Name ____________________ Student No. ____________________

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: 60

[20] 1. (a) State the Pumping Lemma for regular languages.

(b) Show that every finite language is regular, and explain why the Pumping Lemma still holds for finite languages.

Solutions: See the book (or class notes) for part (a). For part (b) notice that if $L = \{w_1, w_2, \ldots, w_n\} \subseteq \Sigma^*$, then if $R = w_1 + w_2 + \cdots + w_n$, i.e., R is a regular expression, then $L(R) = L$. If L is finite, then the Pumping Lemma holds *vacuously* for L, in the sense that we can pick p, the *pumping length*, to be $p = \max\{|w_1|, |w_2|, \ldots, |w_n|\} + 1$.

[20] 2. Show that the language $\{a^n b^n : n \geq 0\}$ is not regular. Note that this is a language over the alphabet $\{a, b\}$.

Solution: If it is regular, then by the Pumping Lemma we can pick a p so that every string longer than p can be pumped. Consider $a^p b^p$; then since $|xy| \leq p$, $y \neq \varepsilon$ is a string of one or more a's. Thus xy^2z is not in the language as it has more a's than b's.

[20] 3. Use the Myhill-Nerode theorem to show that the language $\{ww : w \in \{0,1\}^*\}$ is not regular—note that this is the language of binary strings where the first half equals the second half.

Solution: By the Myhill-Nerode theorem, it is enough to show that there is an infinite sequence of pairwise distinguishable strings; i.e., there is an $x_1, x_2, x_3, \ldots$ such that $x_i \not\equiv_L x_j$ for $i \neq j$, where L is the language in the question.

Let $x_i = ba^i b$; then $x_i a^i \in L$ but $x_j a^i \notin L$ for $i \neq j$.