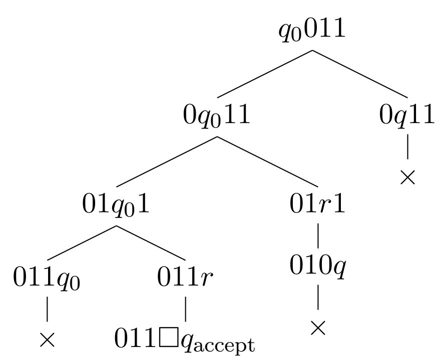
Name ______________________ Student No. ______________________

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: 50

[30] 1. Consider the following nondeterministic Turing machine N presented in class. The transition function of N is given below, together with the "computation tree" on input 011.

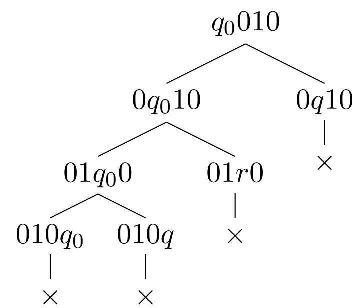
$$
\begin{aligned}
\delta(q_0, 0) &= \{(q_0, 0, \rightarrow), (q, 0, \rightarrow)\} \\
\delta(q_0, 1) &= \{(q_0, 1, \rightarrow), (r, 1, \rightarrow)\} \\
\delta(r, \Box) &= \{(q_{\text{accept}}, \Box, \rightarrow)\} \\
\delta(r, 0/1) &= \{(q, 0, \rightarrow)\}
\end{aligned}
$$



(a) What is the language of N? Justify your answer.
Solution: $L(N) = \{w \in \{0,1\}^* |$ last symbol of w is 1 $\}$.

(b) Present the computation tree of N for 010.

Solution:



(c) Explain, at a high level, how would a deterministic machine M *simulate* the machine N. You can refer to the example of the "computation tree" for 011 in your explanation.

Solution: M maintains a second tape, where it simulates N as follows: it writes the initial configuration C_{init} (in our example q_0011), marks it with an "$*$", and appends to the tape all the configurations that C_{init} yields (in our example $0q_011$ and $0q11$). The $*$ is moved to the next configuration to the right, and the process is repeated with the currently scanned configuration. As soon as an accepting or rejecting configuration is reached, M accepts or rejects accordingly.

[20] 2. A ***Turing machine with left reset*** is similar to an ordinary Turing machine, but the transition function has the form:

$$\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{\rightarrow, \curvearrowleft\}.$$

The difference is that "$\curvearrowleft$" takes the head back to the first square, in one move. So, the head can either move one square to the right, with "$\rightarrow$", or return all the way to the beginning of the tape with "$\curvearrowleft$". That is,

$$\delta(q, a) = (p, b, \curvearrowleft)$$

means that from state q and symbol a, we go to state p and a is overwritten with b, and the head moves all the way back to the first square of the tape.

(a) Define the class of Turing-recognizable languages.

(b) Show that Turing machines with left reset recognize the class of Turing-recognizable languages.

Solutions: For (a), L is Turing-recognizable if there exists a TM M such that

$$L = L(M) = \{w \in \Sigma^* : q_0 w \overset{*}{\Rightarrow} u q_{\text{accept}} v, \text{ for some } u, v \in \Gamma^* \}$$

For (b), we must show that the "left-reset" TMs are capable of simulating ordinary TMs (with left and right transitions).

Let M be an ordinary TM; let M' be the "left-reset" machine that simulates M as follows: if M makes a right-transition, M' does the same. If M makes a left-transition, M' acts as follows: it marks the current position of the head. To place this mark, if the current square contains a $b \in \Gamma$, M' replaces it with $\hat{b}$. The understanding is that $\Gamma_{M'} = \Gamma_M \cup \hat{\Gamma}_M$, where $\hat{\Gamma}$ consists of the "hatted" versions of all the symbols of Γ. Then M' does a left-reset, and moves the contents of each square one position to the right, except for the hatted square, i.e., the "hat" remains in the same tape position. Once all the squares have been shifted one position to the right, M' does a second left-reset, and travels with right-transitions until it reaches the hatted square, where it now does whatever M would have done.

Extra Page