Due: July 6, at 2:10 (in tutorial)

Note: Assignments are due at the *beginning* of tutorial, and late assignments are *not* accepted. The work that you submit *must* be your own. You may discuss problems with each other; however, you should take no notes during these discussions, and you should prepare written solutions alone.

1. The following problems from the textbook (Cormen et al.):
   (a) 27.2-1
   (b) 27.2-3
   (c) 27.2-4
   (d) 27.2-7

2. Recall that the Traveling Salesman Problem (TSP) is: Given n cities $\{1, 2, \ldots, n\}$, and a distance $D(i, j) \in \mathbb{N}$ between each pair (i, j) of cities, find a tour of minimum total distance which starts and ends at city 1, and hits every other city exactly once. The tour can be represented by a sequence $\{1, \pi(2), \pi(3), \ldots, \pi(n), 1\}$, where π is a permutation of the set $\{2, 3, \ldots, n\}$.

   Give a decision problem TSPD corresponding to the Traveling Salesman Problem TSP. Show that TSPD is in NP, and sketch an algorithm showing how to efficiently reduce TSP to TSPD.

3. Let the relation $R(x, y)$ hold iff x and y are finite strings over the alphabet $\{0, 1\}$, and x is an initial segment of y. (In particular, $R(x, y)$ holds if $x = y$.) Give a DTM which recognizes R. Assume that initially the strings x and y appear on the tape, separated by a blank square, and with blanks to the left of x and the right of y, and assume that initially the read/write head scans the left-most square of x (or b, if x is the empty string). Make sure that your DTM halts no matter what input strings x and y are given.

4. If $L \subseteq \Sigma^*$ then we use the notation $\overline{L}$ for the complement of L, that is $\overline{L} = \Sigma^* - L$. Show that if $L \in \mathrm{P}$, then $\overline{L} \in \mathrm{P}$. Argue as follows: Suppose that M is a polytime DTM such that $L(M) = L$. Show how to modify M to form a polytime DTM M' such that $L(M') = \overline{L}$.

5. Let NSP (No Spanning Tree) be the following decision problem:

   Instance: Graph $G = (V, E)$, and edge cost function $c : E \longrightarrow \mathbb{N}$, and a number $B \in \mathbb{N}$.

   Question: Does every spanning tree T of G have cost $c(T) > B$?

   Either show that NSP $\in$ NP, or give a convincing argument that NSP $\notin$ NP.

6. Let ZERO SUM be the following decision problem:

Instance: $d, w_1, \ldots, w_d \in \mathbb{Z}$ (Note that we allow negative weights).

Question: Is there a non-empty subset $S \subseteq \{1, \ldots, d\}$ such that $\sum_{i \in S} w_i = 0$?

Show that ZERO SUM is NP-complete, by doing the following two steps:

(a) Show that ZERO SUM is in NP by giving a "guess and check" algorithm.

(b) Show that PARTITION $\leq_p$ ZERO SUM.