Name ______________________ Student No. ______________________

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: 40

1. Recall that the decision problem k-COLOR is defined as follows:
   Input: A graph $G = (V, E)$.
   Question: Can the nodes of G be colored using at most k colors so that adjacent nodes have different colors?

[6] (a) Prove that 2-COLOR is in P.

[4] (b) We know that 3-SAT is NP-complete. Assuming that P $\neq$ NP, explain very briefly why it *is not* true that 3-SAT $\leq_p$ 2-COLOR.

[4] (c) We know that 3-COLOR is NP-complete. Explain, very briefly, why it *is* true that 3-SAT $\leq_p$ 3-COLOR.

2. Recall that VC is an NP-complete decision problem defined as follows:
Input: Graph $G = (V, E)$, positive integer B.
Question: Does G have a vertex cover of size B?

Let ODD-VC be the same as VC, except that B must be odd. Show that ODD-VC is NP-complete as follows:

[3] (a) Show that ODD-VC is in NP.

[6] (b) Show that VC $\leq_p$ ODD-VC. State the correctness condition of your reduction. (You do not have to prove the correctness condition).

3. For each of the following assertions, state whether it is true or false, and justify it. If it is true, justify it with a proof; if it is false, justify it with a counter-example. (No marks for a guess without justification).

[4] (a) For all languages L_1 and L_2, if $L_1 \leq_m L_2$ and L_2 is semi-decidable, then L_1 is also semi-decidable.

Circle one: True False

Justification:

[4] (b) For all languages L_1 and L_2, if $L_1 \subseteq L_2$ and $L_2 \in \mathrm{P}$, then $L_1 \in \mathrm{P}$.

Circle one: True False

Justification:

[4] (c) For all languages L, if both L and $\overline{L}$ are semi-decidable, then L is decidable.

Circle one: True False

Justification:

[5] (d) Let L be a language. Suppose that $L = L(E)$ where E is an enumerator that outputs the strings of L in order of their length, i.e. it outputs first all the strings of length 1 (if any), then all the strings of length 2 (if any), and so on. Then, L is decidable.

Circle one: True False

Justification: