CSC 364H **Problem Set 3** **Summer 99**

Due: day section: August 4, at 2:10 (in tutorial)
evening section: August 3, at 6:10 (in tutorial)

Note: Assignments are due at the *beginning* of tutorial, and late assignments are *not* accepted. The work that you submit *must* be your own. You may discuss problems with each other; however, you should take no notes during these discussions, and you should prepare written solutions alone.

1. Each of the following decision problems is either in P or it is NP-complete. State which and justify *briefly*. To justify " in P ", outline a polytime algorithm, or explain how to use or modify a polytime algorithm given in class.

   To justify " NP-complete ", explain *briefly* why the problem is in NP, state which standard NP-complete problem can be reduced to the given problem, and describe the reduction. For standard NP-complete problems, you may use any of the following problems (described in class):

   SAT
   3SAT
   VC (Vertex Cover)
   CLIQUE
   IS (Independent Set)
   PARTITION
   SKD
   HC (Hamiltonian Cycle) and HP (Hamiltonian Path)
   3-COLOUR

   (a) SMOOTH SAT
   This is the same as SAT, except that each input clause C_i must contain only negated variables (of the form $\bar{P}$) or only unnegated variables. $P \vee Q$ and $\bar{P} \vee \bar{Q}$ are OK, but $P \vee \bar{Q}$ is not OK.

   (b) LARGE IS
   This is the same as IS, except that the number $n = |V|$ of vertices must be even, and the input parameter B must be exactly $n/2$.

   (c) DEGREE 2 IS
   This is the same as IS, except that each vertex of the input graph G must have degree 2.

   (d) UNARY PARTITION
   This is the same as PARTITION, except that the input numbers $w_1, \ldots, w_d$ are presented in unary notation (i.e. a string of 1s).

(e) DEGREE 3 CLIQUE
This is the same as CLIQUE, except that each node of the input graph G must have degree 3.

(f) HIGH PRECISION 2-PROCESSOR SCHEDULING
Input: We wish to schedule n jobs $\{1, \ldots, n\}$ on two processors P and P'. Job i takes time t_i on processor P and time t'_i on processor P'. We are given positive integers $t_1, \ldots, t_n, t'_1, \ldots, t'_n$ and T, all in binary notation (representing times in nanoseconds). Each processor executes one job at a time.

Question: Can each of the n jobs be assigned to either P or P' so that both processors finish all their assigned jobs within time T?

(g) LARGE CYCLE
Input: Graph $G = (V, E)$, positive integer B.

Question: Does G have a cycle with B or more distinct nodes?

(h) SMALL CYCLE
Input: Graph $G = (V, E)$, positive integer B.

Question: Does G have a cycle with B or fewer nodes?

(i) 4-COLOUR
Input: Graph $G = (V, E)$.

Question: Can the nodes of G be coloured using at most 4 colours such that adjacent nodes have different colours?

(j) TRIPARTITE
Input: Graph $G = (V, E)$.

Question: Can V be written as a union $V = V_1 \cup V_2 \cup V_3$ of three sets, such that each V_i is an independent set in G.

(k) s-t HP
Is the same as HP, except that the input specifies not only the graph G, but also two specific nodes in G, s and t, and the question is whether there is a Hamiltonian Path in G which starts at s and ends at t.

2. Define the problem PARTITION-SEARCH as follows:

Input: $\langle d, w_1, \ldots, w_d \rangle$ where $d, w_i \in \mathbb{N}$.

Output: A set S such that $S \subseteq \{1, \ldots, d\}$ and $\sum_{i \in S} w_i = \sum_{i \in \bar{S}} w_i$.

Prove that PARTITION-SEARCH $\xrightarrow{p}$ PARTITION.

3. The following problems from pages 148 and 149 of Sipser's book (page 159 in the course readings):

(a) Problem 3.6
(b) Problem 3.11
(c) Problem 3.14a,d,e
(d) Problem 3.16