CSC 364H (L5101) **Test 1 – solutions** **Summer 99**

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: 30

[6] 1. Sort the jobs in non-decreasing order of finish times, so that $e_1 \leq e_2 \leq \cdots \leq e_n$, and consider the jobs in order, adding each job if it does not start before the finish time of the last job added.

[12] 2. (a) Initialization:
$R(0,0) = T$, $R(0,j) = F$ for all $j > 0$.

(b) For $i > 0$, $R(i,j) = T$ iff
$R(i-1,j) = T$ or ($j \geq w_i$ and $R(i-1, j-w_i) = T$).

(c) Justification: Suppose we do not include object i. Then, obviously, $R(i,j) = T$ iff $R(i-1,j) = T$. Suppose object i is included. Then it must be the case that $R(i,j) = T$ iff $R(i-1, j-w_i) = T$, $j - w_i \geq 0$ (i.e., there is a subset S of objects $\{1, ..., i-1\}$ s.t. the sum of weights $K(S)$ is exactly $j - w_i$, $j \geq w_i$).

[12] 3. (a) Initialization:
$A(0,t) = 0$, $0 \leq t \leq t_n$

(b) To compute $A(i,t)$ for $i > 0$ first define $t_{\min} = \min\{t, t_i\}$. Now

$$A(i,t) = \begin{cases} A(i-1,t) & \text{if } t_{\min} < d_i \\ \max\{A(i-1,t), p_i + A(i-1, t_{\min} - d_i)\} & \text{otherwise} \end{cases}$$

(c) Justification: If job i is scheduled in the optimal schedule, it finishes at time $t_{\min}$, and starts at time $t_{\min} - d_i$. If it is scheduled, the maximum possible profit is $A(i-1, t_{\min} - d_i) + p_i$. Otherwise, the maximum profit is $A(i-1,t)$.