CSC 364H (L0101) **Test 2 — solutions** **Summer 99**

1. (a)

$$E = \begin{pmatrix} x_{11} & 0 & x_{13} \\ x_{21} & x_{22} & 0 \\ 0 & x_{32} & x_{33} \end{pmatrix}$$

   (b) $\det(E) = x_{11}x_{22}x_{33} + x_{13}x_{12}x_{32}$

   (c) G has 2 perfect matchings, and they are:

$$\{(1,1),(2,2),(3,3)\}$$
$$\{(1,3),(2,1),(3,2)\}$$

   (d) $x_{11} = x_{22} = x_{33} = x_{13} = x_{21} = 1$, and $x_{32} = -1$.

   (e) Suppose $r_{11}, r_{22}, r_{33}, r_{13}, r_{21}$ have been chosen. Then, the choice of r_{32} will decide if $r_{11}r_{22}r_{33} = r_{13}r_{21}r_{32}$, in which case $\det(M') \neq 0$, or if $r_{11}r_{22}r_{33} = -r_{13}r_{21}r_{32}$, in which case $\det(M') = 0$. Since $\Pr(r_{32} = 1) = \Pr(r_{32} = -1) = 1/2$, it follows that $\Pr(\det(M') = 0) = 1/2$.

2. (a) Given I, an instance of PARTITION, we want to compute in polytime I', an instance of SKD, such that I is a "YES" instance iff I' is a "YES" instance.

   Let $I = \langle d, w_1, \dots, w_d \rangle$. Then, $I' = \langle d', w'_1, \dots, w'_{d'}, C, B \rangle$, where $d = d'$, $w_i = w'_i$ for $1 \leq i \leq d' = d$, and $C = B = 1/2 \sum_{i=1}^{d} w_i$.

   (b) 3SAT is NPC (we showed this in class). So, by the NPC Lemma, PARTITION is NPC. Since PARTITION $\leq_p$ SKD (part (a)), by the NPC Lemma again, SKD is NPC.

   We can also prove this using the Transitivity Lemma: since 3SAT $\leq_p$ PARTITION $\leq_p$ SKD, by the Transitivity Lemma 3SAT $\leq_p$ SKD, so by the NPC Lemma, SKD in NPC.

3. (a) `...bb010bb...`

   (b) q_{reject}
   q_5, print b, left
   q_0, print b, right