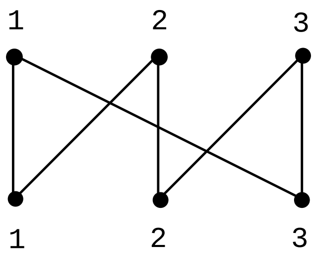
CSC 364H (L0101) **Test 2** **Summer 99**

Name ______________________ Student No. ______________________

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: 30

1. Consider the bipartite graph G:



(a) Complete the Edmonds matrix E of G.

$$E = \begin{pmatrix} x_{11} & 0 & x_{13} \\ & & \\ & & \end{pmatrix}$$

(b) What is $\det(E)$?

$\det(E) =$

(c) How many perfect matchings does G have? List each perfect matching.

(d) Let $S = \{-1, 1\}$. Let M' be the result of substituting members of S randomly and independently for the variables in M, where each member of S is equally likely to be substituted for each variable. Give an example of a possible M' such that $\det(M') = 0$.

(e) Find exactly the probability that $\det(M') = 0$, where M' is chosen randomly as above. Justify your answer briefly.

2. Recall the following two problems:

SKD (Simple Knapsack *Decision*):
Input: $d, w_1, \dots, w_d, C, B \in \mathbb{N}$, presented in binary.
Question: Is there a subset $S \subseteq \{1, \dots, d\}$ such that $B \leq \sum_{i \in S} w_i \leq C$?

PARTITION:
Input: $d, w_1, \dots, w_d \in \mathbb{N}$, presented in binary.
Question: Is there a subset $S \subseteq \{1, \dots, d\}$ such that $\sum_{i \in S} w_i = \sum_{i \in \bar{S}} w_i$?

(a) Show that PARTITION $\leq_p$ SKD by providing a polytime reduction. (You don't have to prove that your reduction is correct).

(b) Suppose that you succeed in proving that 3SAT $\leq_p$ PARTITION. Is this enough to conclude that SKD is *NP*-complete? Explain briefly.

3. Consider the language $L = \{0^i10^j \mid 1 \leq i \leq j\}$. For example, the strings 010,0100,00100 are all in L, but the strings 10, 0010, 001100 are not.

Below is a partially specified Turing Machine M which is supposed to recognize L.

(a) Suppose the input string is 00100, so the initial tape configuration is:

...bb$\underline{0}$0100bb...

and the head is reading the underlined symbol. Follow the computation of M on this input until M reaches state q_4 and the head is scanning the symbol 1. Give the resulting tape configuration below:

(b) Complete the three indicated lines below by giving the action of M in each case:

	state	symbol	action
	q_0	0	q_1, print b, right
(input must start with 0)	q_0	b,1	q_{reject}
(go right to 1)	q_1	0	q_1, print 0, right
	q_1	1	q_2, print 1, right
	q_1	b	
(go right to b)	q_2	0	q_2, print 0, right
	q_2	b	q_3, print b, left
(two 1s)	q_2	1	q_{reject}
(cross out 0)	q_3	0	q_4, print b, left
(not enough zeroes to right)	q_3	1	q_{reject}
(go left to 1)	q_4	0	q_4, print 0, left
	q_4	1	q_5,
	q_5	b	q_{accept}
	q_5	0	q_6, print 0, left
(go left to b)	q_6	0	q_6, print 0, left
	q_6	b	