1. (a) Choose r uniformly at random from the set $\{1, 2, \ldots, 2d\}$, and output "YES" if $a(r) \cdot b(r) = c(r)$, and "NO" otherwise.

   (b) If $a(x) \cdot b(x) = c(x)$, then for any r, $a(r) \cdot b(r) = c(r)$. Thus the algorithm will always answer correctly in this case, so the probability of error is 0.

   (c) The probability of error in this case is the probability that $a(r) \cdot b(r) = c(r)$ given that $a(x) \cdot b(x) \neq c(x)$. So the probability of error is

   $$\Pr[a(r) \cdot b(r) - c(r) = 0 | a(x) \cdot b(x) - c(x) \neq 0] \leq d/2d = 1/2$$

   by the Schwartz-Zippel Theorem.

   (d) Invoke the randomized algorithm k times on the same input, and output "YES" iff the randomized algorithm answers "YES" every time.

2. (a) We have to provide a polytime "guess-and-check" algorithm: *guess* a truth assignment t to the variables in S, and *check* that t satisfies S.

   (b) Given an instance $S = \{C_1, \ldots, C_m\}$ of 4-SAT, we want to compute an instance $S' = \{C'_1, \ldots, C'_{m'}\}$ of 3-SAT, such that S is satisifiable iff S' is satisfiable.

   For every clause $C = L_1 \vee L_2 \vee L_3 \vee L_4$ in S, replace it by $C' = P \vee L_3 \vee L_4$ (where P is a *new* variable), and add the following new clauses

   $$\begin{aligned} D_1 &= \bar{P} \vee L_1 \vee L_2 \\ D_2 &= P \vee \bar{L}_1 \vee Q \\ D'_2 &= P \vee \bar{L}_1 \vee \bar{Q} \\ D_3 &= P \vee \bar{L}_2 \vee Q \\ D'_3 &= P \vee \bar{L}_2 \vee \bar{Q} \end{aligned}$$

   where Q is any variable.

3. (a) `...bb101bb111bb...`

   (b) q_{accept}

   q_{reject}

   $q_4, \text{print b, left}$