CSC 364H (L5101) **Test 2** **Summer 99**

Name ______________________ Student No. ______________________

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: 30

1. Define POLYEQ to be the following problem:
   Input: $\langle d, a(x), b(x), c(x) \rangle$, where $a(x), b(x)$ and $c(x)$ are polynomials in $\mathbb{Q}[x]$, i.e. polynomials with rational coefficients, and $d = \text{degree}(a) + \text{degree}(b) = \text{degree}(c)$.
   Output: "YES" if $a(x) \cdot b(x) = c(x)$, and "NO" otherwise.

   (a) Describe a Randomized polytime Algorithm for this problem with error probability $\leq 1/2$.

   (b) Estimate the probability that your algorithm gives the wrong answer in the case that the correct answer is "YES" (i.e. your algorithm answers "NO" even though $a(x) \cdot b(x) = c(x)$). Justify briefly.

(c) Estimate the probability that your algorithm gives the wrong answer in the case that the correct answer is "NO" (i.e. your algorithm answers "YES" even though $a(x) \cdot b(x) \neq c(x)$). Justify briefly.

(d) Explain how to improve your algorithm so that it is still polytime, but the probability of error is $\leq \frac{1}{2^k}$, where k is a parameter provided with the input.

2. k-SAT is defined as follows:

Input: A set $S = \{C_1, \dots, C_m\}$ of clauses, such that every clause has exactly k literals.
Question: Is S satisfiable?

(a) Show that k-SAT is in *NP* for every $k \in \mathbb{N}$.

(b) Show that 4-SAT $\leq$ 3-SAT by providing a polytime reduction. (You don't have to prove that your reduction is correct).

3. Below is a partially specified Turing Machine M which is supposed to take as input two numbers x and y written in binary notation and determine whether $x \le y$. Assume that the initial tape configuration is $\bar{x}b\bar{y}$, where $\bar{x}$ and $\bar{y}$ are the binary notation for x and y, respectively. Assume that $\bar{x}$ and $\bar{y}$ have the *same length* ($\bar{x}$ and $\bar{y}$ may have leading zeores, i.e. begin with 0s).

   (a) Suppose that the initial tape configuration is:

   `...bb`<u>`0`</u>`101b0111bb...`

   and the head is reading the underlined symbol. Follow the computation of M on this input until M reaches state q_0 for the second time (that is, the first time it reaches q_0 after starting in state q_0). Give the resulting tape configuration below:

   (b) Complete the three lines indicated by giving the action of M in each case:

	state	symbol	action
	q_0	b	q_{accept}
	q_0	0	q_1, print b, right
	q_1	b	q_{accept}
	q_1	0,1	q_2, print 0,1 (respectively), right
(go right to b)	q_2	0,1	q_2, print 0,1 (respectively), right
	q_2	b	q_3, print b, right
(go right to 0 or 1)	q_3	b	q_3, print b, right
	q_3	0	q_4, print b, left
	q_3	1	
(go left to 0 or 1)	q_4	b	q_4, print b, left
	q_4	0,1	q_5, print 0,1 (respectively), left
(go left to b)	q_5	0,1	q_5, print 0,1 (respectively), left
	q_5	b	q_0, print b, right
	q_0	1	q_6, print b, right
	q_6	0,1	q_7, print 0,1 (respectively), right
(last run)	q_6	b	q_9, print b, right
(go right to b)	q_7	0,1	q_7, print 0,1 (respectively), right
	q_7	b	q_8, print b, right
(go right to 0 or 1)	q_8	b	q_8, print b, right
	q_8	0	
	q_8	1	
(go right to 0 or 1)	q_9	b	q_9, print b, right
	q_9	0	q_{reject}
	q_9	1	q_{accept}