CSC 364H (L0101) **Test 3** **Summer 99**

Name ______________________ Student No. ______________________

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: 30

1. Recall that k-COLOUR is defined as follows:
   Input: A graph $G = (V, E)$.
   Question: Can the nodes of G be coloured using at most k colours so that adjacent nodes have different colours?

[6] (a) Prove that 2-COLOUR is in P.

[4] (b) We know that 3-COLOUR is NP-complete. Assuming that $P \neq NP$, show that it is not true that 3-COLOUR $\leq_p$ 2-COLOUR.

2. Recall that IS is a decision problem defined as follows:
Input: Graph $G = (V, E)$, positive integer B.
Question: Does G have an independent set of size B?

Let EVEN IS (EIS) be the same as IS, except B must be even. Show that EIS is NP-complete as follows:

[2] (a) Show that EIS is in NP, by giving a suitable guess-and-check algorithm.

[6] (b) Show that IS $\leq_p$ EIS by giving a polytime reduction. State the correctness condition of your reduction. (You do not have to prove the correctness condition).

3. Prove or disprove each of the following assertions. (To disprove, give a specific example).

[4] (a) For all languages L_1 and L_2, if $L_1 \leq_m L_2$ and L_2 is semi-decidable, then L_1 is also semi-decidable.

[4] (b) For all languages L_1 and L_2, if $L_1 \subseteq L_2$ and $L_2 \in P$, then $L_1 \in P$.

[4] (c) For all languages L, if both L and $\overline{L}$ are semi-decidable, then L is decidable.