CSC 364H (L5101) **Test 3** **Summer 99**

Name ______________________________ Student No. ______________________________

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: 30

[4] 1. Recall the decision problem CLIQUE:
Input: Graph $G = (V, E)$, positive integer B.
Question: Does G have a clique of size B?

Suppose that you succeed in finding a dynamic programming algorithm that solves CLIQUE in polytime. What, if any, are the implications for solving the search problem SKS (Simple Knapsack Search)? Explain briefly.

Hint: You may assume that CLIQUE is NP-complete.

2. Let EVEN CLIQUE be the same as CLIQUE, except that B must be even. Recall that, if $G = (V, E)$ is a graph, then $V' \subseteq V$ is a clique in G if every pair of distinct nodes in V' is connected by an edge in E.

   Show that EVEN CLIQUE is NP-complete as follows:

[2] (a) Show that EVEN CLIQUE is in NP.

[2] (b) To show that EVEN CLIQUE is NP-hard, state what you are reducing to what.

[2] (c) State explicitly (in terms of graphs and cliques) the correctness condition for the reduction.

[5] (d) Give the reduction.

[5] (e) Prove the correctness condition.

3. Prove or disprove each of the following assertions. (To disprove, give a specific example).

[5] (a) For all languages L_1 and L_2, if $L_1 \subseteq L_2$ and L_2 is semi-decidable, then L_1 is also semi-decidable.

[5] (b) Let L be a language. Suppose that $L = L(E)$ where E is an enumerator that outputs the strings of L in order of their length, i.e. it outputs first all the strings of length 1 (if any), then all the strings of length 2 (if any), and so on. Then, L is decidable.