Solutions to Assignment 1 (30 marks)

2.2.5

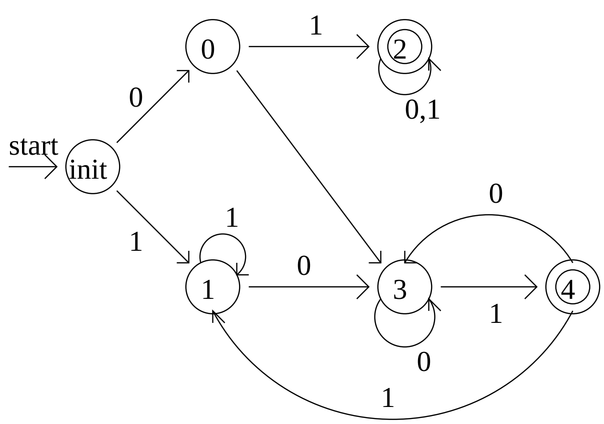
(a)[5] Let $Q = \{q_{x_1 \ldots x_i} |$ where $1 \leq i \leq 5$ and $x_j \in \{0,1\}$ $\} \cup \{q_{init}\} \cup \{q_{reject}\}$.
Let $F = Q - \{q_{reject}\}$ (i.e., F contains everything, but the q_{reject} state; this implies, that we accept all the strings of length < 5).
Finally, let $\delta(q_{init}, 0/1) = q_{0/1}$ and $\delta(q_{reject}, 0/1) = q_{reject}$, and let:

$$\delta(q_{x_1 \ldots x_i}, 0/1) = \begin{cases} q_{x_1 \ldots x_i 0/1} & \text{if } i < 4 \\ q_{x_1 \ldots x_4 0/1} & \text{if } x_1 \ldots x_4 0/1 \text{ has two zeros} \\ q_{reject} & \text{if } x_1 \ldots x_4 0/1 \text{ does } \textit{not} \text{ have two zeros} \\ q_{x_2 \ldots x_5 0/1} & \text{if } x_2 \ldots x_5 0/1 \text{ has two zeros} \\ q_{reject} & \text{if } x_2 \ldots x_5 0/1 \text{ does } \textit{not} \text{ have two zeros} \end{cases}$$

(b)[5] $Q = \{q_{x_1 \ldots x_i} | 1 \leq i \leq 10, x_j \in \{0,1\}\} \cup \{q_{init}\}$.
$F = \{q_{1 x_2 \ldots x_{10}} | x_j \in \{0,1\}\}$.
$\delta(q_{init}, 0/1) = q_{0/1}$, and:

$$\delta(q_{x_1 \ldots x_i}, 0/1) = \begin{cases} q_{x_1 \ldots x_i 0/1} & \text{if } i < 10 \\ q_{x_2 \ldots x_{10} 0/1} & \text{if } i = 10 \end{cases}$$

(c)[5]



(d)[5] $Q = \{q_{xy} | x \in \{0,1,2,3,4\}, y \in \{0,1,2\}\}$, where q_{00} is the initial state, and also the only accepting state; that is, $F = \{q_{00}\}$. The idea is that q_{xy} represents that the nr of 0s is divisible by x and the nr of 1s is divisible by y. The transition function is given as follows:

$$\delta(q_{xy}, 0) = q_{(x+1 \bmod 5)y}$$
$$\delta(q_{xy}, 1) = q_{x(y+1 \bmod 3)}$$

For example, $3 + 1 \mod 5 = 4$ but $4 + 1 \mod 5 = 0$.

2.2.7[10] The Basis Case is $w = \varepsilon$, so by definition $\hat{\delta}(q, \varepsilon) = q$. In the Induction Step, let $w = xa$, then $\hat{\delta}(q, w) = \hat{\delta}(q, xa) = \delta(\hat{\delta}(q, x), a)$, by definition of $\hat{\delta}$. By the Induction Hypothesis, $\hat{\delta}(q, x) = q$, and $\delta(q, a) = q$, so we are done.