Solutions to Assignment 2 (50 marks)

4.1.1[30]
(a)[5] Done in class.
(b)[5] Suppose that this language is regular. Then there is an n as in the "Pumping Lemma" (PL). Choose w to be $(^n)^n$, so that $|w| = 2n$. Then, no matter how we choose x, y, z, $y = (^i$ for some $i > 0$. By pumping y we unbalance the number of left and right parenthesis, so xy^kz is not well formed (for $k > 1$).
(c)[5] Let n be as in the PL. Choose $w = 0^n10^n$, so no matter how we choose x, y, z, $y = 0^i$, for $i > 0$. By pumping y we get more zeros on the left than on the right.
(d)[5] Let n be as in the PL. Choose $w = 0^n12^n$, so $y = 0^i$, for some $i > 0$. By pumping y we get more zeros than twos.
(e)[5] Question 3 on Test 1.
(f)[5] Let n be as in the PL. Choose $w = 0^n1^{2n}$. Then $y = 0^i$ for some $i > 0$. Now pump y, to get xy^kz, where k is any integer such that $ki > 2n$. Then we get more zeros than ones, so the result is certainly not of the form 0^m1^{2m} for some m.

4.3.3[10] Since L is a regular language, there exists a DFA D such that $L = L(D)$. Now, let D' be D, but where $F' = Q - F$. Then, $L(D) = \Sigma^* \iff L(D') = \emptyset$. We already know how to test for emptiness: $L(D') = \emptyset$ iff there is no path in the DFA D' from the initial state to some accepting state (reachability algorithm).

4.3.4[10] L_1, L_2 have at least one string in common iff $L_1 \cap L_2 \neq \emptyset$. Since L_1, L_2 are both regular, so is $L_1 \cap L_2$ by Theorem 4.8. We again use the reachability algorithm on the DFA recognizing $L_1 \cap L_2$ to test for emptiness.