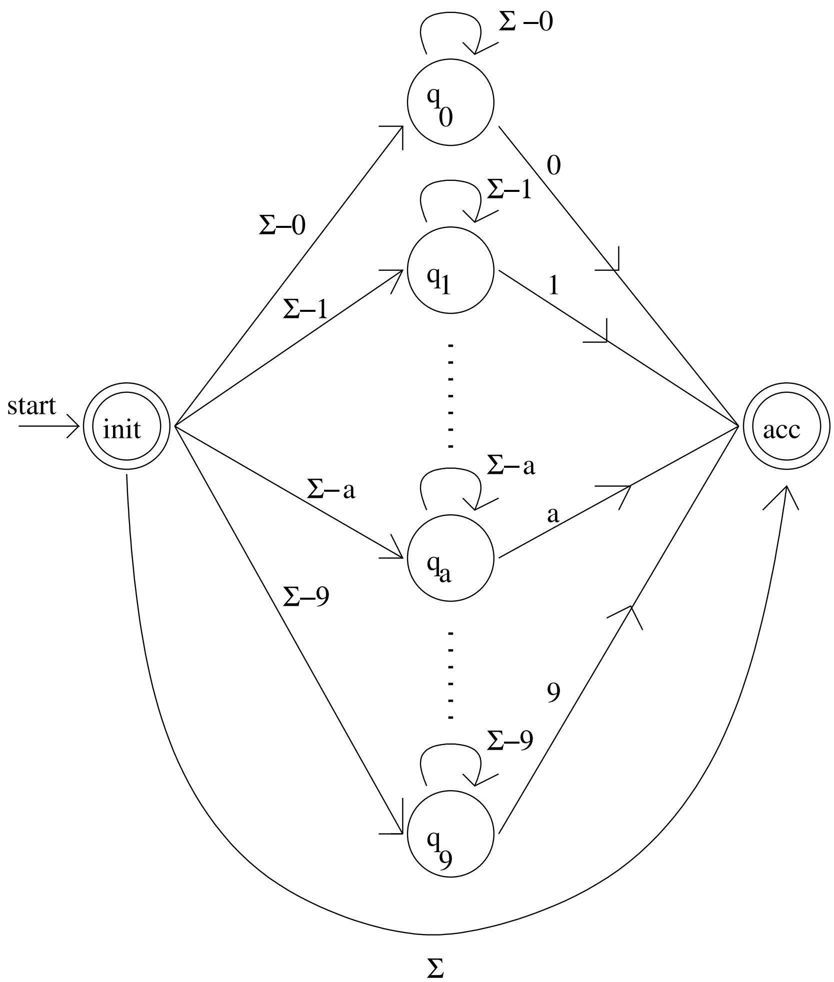
Name ______________________ Student No. ______________________

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: 30

[10] 1. Give a non-deterministic finite automaton (an NFA) that accepts the following language L: the set of strings over the alphabet $\{0, 1, 2, \ldots, 9\}$ such that the final digit has *not* appeared before.

Thus, $092338, 783224 \in L$, but $129892 \notin L$.



[5] 2. (a) Write a regular expression for the following language: the set of strings over the alphabet $\{a, b, c\}$ containing at least one a and at least one b.

$$(a+b+c)^*(a(a+b+c)^*b + b(a+b+c)^*a)(a+b+c)^*$$

[5] (b) Prove or disprove the following statement about regular expressions:

$$(R+S)^* = R^* + S^* \tag{1}$$

(State clearly if you are proving or disproving.)

We will disprove it, using "the Test" which states that we can replace R, S by concrete symbols a, b, respectively and consistently throughout the equation, and equation (1) holds iff:

$$(a+b)^* = a^* + b^* \tag{2}$$

holds. Now, we can show that equation (2) fails to hold with a counter-example: $ab \in L((a+b)^*)$, but $ab \notin L(a^* + b^*)$.

[10] 3. Prove that the following language L is *not* regular: L is the set of strings over the alphabet $\{0, 1\}$ consisting of a block of p 0s followed by a block of q 1s, where $p \le q$, that is,

$$L = \{0^p 1^q | p \le q\}$$

So for example, $00111, 01 \in L$, but $00011, 0011101 \notin L$.

We use the "Pumping Lemma" (PL) to show that L is not regular; we argue by contradition: suppose that L is regular. Then, by the PL, there exists a positive integer n such that:

$\forall w \in L$ such that $|w| \ge n$, there exist x, y, z with the following properties: $w = xyz$, and:

(a) $|xy| \le n$

(b) $y \ne \varepsilon$

(c) $xy^k z \in L$ for all $k \ge 0$

Choose $w = 0^n 1^n$. Since $|w| \ge n$, and $w \in L$, it follows that we can choose x, y, z as above. By (a), $|xy| \le n$, and since by (b) $y \ne \varepsilon$, it follows that $y = 0^i$, for some $i \ge 1$. Finally, by (c), $xy^k z \in L$ for all $k \ge 0$, so choose $k = 2$ (for example, but any $k \ge 2$ would do), then $xy^k z$ has more zeros than ones, so it is not in L!

Contradiction. It follows that our original assumption that L is regular is false.