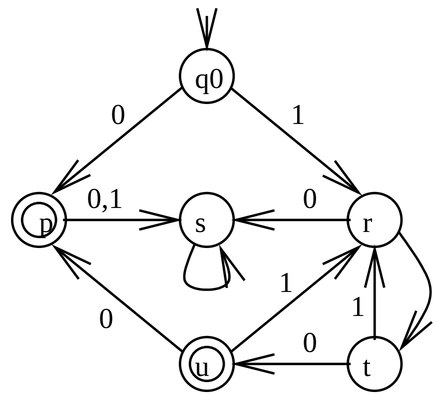
Assignment 2–Solutions **SE4I03 Fall 2002** **due on Oct. 11**

1. Here is the DFA accepting $(11 + 110)^*0$:



2. L is the set of strings of 0s and 1s such that at least the first half of the string consists of 0s. Assume that L is a regular language. Let $w = 0^n1^n$, where n is the integer in the Pumping Lemma. Since $w = xyz$, and $|xy| \leq n$ and $y \neq \varepsilon$, it follows that $y = 0^i$, for some $1 \leq i \leq n$. On the other hand, for all $k \geq 0$, we must have that $xy^kz \in L$. Let $k = 0$, so $y^k = (0^i)^0 = \varepsilon$, so $xy = 0^{n-j}$, so $xy^0z = 0^{n-j}1^n$ which is not in L; contradiction. Thus L is not regular.

3. Let n be the integer in the Pumping Lemma for L. Let $w = xyz$, as in the statement of the Lemma. Let $p = |x| + |z|$ and let $q = |y| > 0$ (since $y \neq \varepsilon$). Since xy^mz is in L for all $m \geq 0$, and $|xy^mz| = |x| + |y^m| + |z| = |x| + |z| + m|y| = p + mq$, we are done.

4. $L = \{0^2, 0^3, 0^5, 0^7, 0^{11}, \ldots\}$. By Theorem 1 above, to show that L is not regular, it is enough to show that the set of primes cannot contain an infinite arithmetic progression of the form $\{p + mq | m \geq 0\}$. Thus, we must show that for any $p \geq 0$, and $q > 0$, there is an m such that $p + mq$ is not a prime number.

   Not being prime means that $p+mq$ must have a factor 2 or bigger. Thus, letting $m = p$ to obtain $p+mq = p+pq = p(1+q)$ is not good enough, since we only know that $p \geq 0$, so p could be 1. On the other hand, choose $m = p + 2q + 2$ works nicely, since in that case, $p + mq = p + pq + 2q^2 + 2q = (p + 2q) + (p + 2q)q = (p + 2q)(1 + q)$. Since $q > 0$, $1 + q \geq 2$, so this is our factor.