Name ______________________ Student No. ______________________

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: 50

[10] 1. Consider the following alphabet $\Sigma = \{1, 2, 5\}$, and let L be the language:

$$L = \{w | w = w_1 w_2 \ldots w_n,\ w_i \in \Sigma \text{ and } w_1 + w_2 + \cdots + w_n = 10\}$$

So for example $11125 \in L$ as $1 + 1 + 1 + 2 + 5 = 10$, and $22222 \in L$, but $155 \notin L$.

Is L a regular language? Justify your answer.

Solution: L is a regular language. The simplest justification is that there are finitely many strings over Σ whose digits add up to 10, and hence L is finite, and all finite languages are regular.

2. Let $L_{\text{twice}} = \{0^n 1^{2n} | n \geq 0\}$, so $L_{\text{twice}} = \{\varepsilon, 011, 001111, 000111111, \ldots\}$.

[5] (a) Show that L_{twice} is not a regular language using the *Pumping Lemma.*

Solution: Assume L_{twice} is regular. By the PL, there is a constant n such that for all w, $|w| \geq n$, we can find x, y, z such that $w = xyz$, and (i) $y \neq \varepsilon$, (ii) $|xy| \leq n$, and (iii) $\forall k \geq 0$, $xy^k z \in L_{\text{twice}}$. Consider $0^n 1^{2n}$. By (i) and (ii) we know that $y = 0^j$, $j > 0$. By (iii) (with $k = 2$) we conclude that $0^{n-j}(0^j)^2 1^{2n} \in L_{\text{twice}}$, which is a contradiction, as we have $n + j$ zeros ($j > 0$) and $2n$ ones. Therefore, L_{twice} is not regular.

[5] (b) Show that L_{twice} is not a regular language using the *Myhill-Nerode Theorem.*

Solution: L_{twice} is not regular because all strings of the form 0^i, with $i \geq 0$, are distinguishable: for $i < j$, 0^i and 0^j are distinguished by 1^{2i}, because $0^i 1^{2i} \in L_{\text{twice}}$, but $0^j 1^{2i} \notin L_{\text{twice}}$. Thus, $[0^i]$ are distinct equivalence classes for all i. Therefore, L_{twice} has infinitely many equivalence classes, and hence it is not regular by the M-N Theorem.

[10] 3. Suppose that L is a regular language with alphabet Σ, and given by a DFA A, i.e., $L(A) = L$.

Give an algorithm to tell whether $L = \Sigma^*$. That is, give an algorithm for testing whether L contains all the strings over Σ.

Solution: Let A be the DFA recognizing L. Let $\overline{A}$ be A with the accept and reject states interchanged (hence $\overline{A}$ recognizes the complement of L, $\overline{L}$). Now test for the emptiness of $\overline{L}$ be checking if there is a path in $\overline{A}$ from the initial state to some accepting state. $\overline{L}$ is empty iff $L = \Sigma^*$.

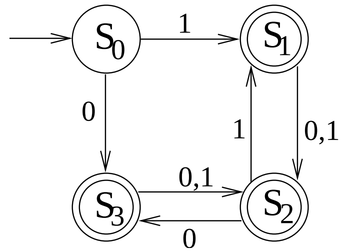
[10] 4. Given a DFA A, we say that the states p and q are *equivalent* (with respect to A), if:

For all w, $\hat{\delta}(p, w)$ is an accepting state iff $\hat{\delta}(q, w)$ is an accepting state.

Show that this relation of states is indeed an *equivalence relation*.

Solution: Reflexivity and Symmetry are immediate. Transitivity is Theorem 4.23 in the text-book.

[10] 5. Consider the DFA A given by:



Find the minimal DFA equivalent to it.

Solution: The minimal DFA has two states, $[s_0]$ and $[s_1]$, and and arrow labeled with $0, 1$ going from $[s_0]$ to $[s_1]$, and a self-loop labeled with $0, 1$ at $[s_1]$.