Software Engineering 4I03 — Deferred Exam Michael Soltys

DAY CLASS
DURATION OF EXAMINATION: 3 Hours
MCMASTER UNIVERSITY FINAL EXAMINATION April 2005

THIS EXAMINATION PAPER INCLUDES 1 PAGE AND 10 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

No aids allowed. All questions worth 10 marks, for a total of 100.

1. Explain why nondeterministic push-down automata accept the same class of languages, regardless of whether they accept by final state of by empty stack. Does the same hold true for deterministic push-down automata? Explain your answer.
2. Show that for every $n \geq 1$ (n a natural number), there exists a language $L_n \subseteq \{0,1\}^*$ such that L_n is accepted by an NFA with $O(n)$ states, while every DFA that accepts L_n requires $O(2^n)$ many states. ($O(n)$ stands for Big-Oh notation.)
3. Consider the following context-free grammar G: $S \longrightarrow aS|Sb|a|b$. (a) Prove by induction on the string length that no string in $L(G)$ has ba as a substring. (b) Describe $L(G)$ informally; justify your answer using part (a).
4. Show that $L = \{0^n 1^m | m \leq n^3\}$ is not a context-free language.
5. The following grammar generates *prefix* expressions with operands x and y and binary operations $+, -, *$: $E \longrightarrow +EE| * EE| - EE|x|y$. (a) Find leftmost and rightmost derivations, and a derivation tree for `+*-xyxy`. (b) Prove that this grammar is unambiguous.
6. Give a convention for assigning a unique number to every Turing machine. Then, use your convention to give a number to the Turing machine which on every input enters immediately an accepting state and halts.
7. Let $L_1, L_2, \ldots, L_k$ be a collection of languages over some alphabet Σ such that: (i) for all $i \neq j$, $L_i \cap L_j = \emptyset$, (ii) $L_1 \cup L_2 \cup \ldots \cup L_k = \Sigma^*$, and (iii) all languages are recursively enumerable. Prove that each language is therefore recursive.
8. State the Post Correspondence Problem, and explain (at a high level) why is it undecidable.
9. Given a context-free grammar G, is the problem of deciding whether $L(G) = \Sigma^*$ (i.e., whether G generates all strings) decidable? Explain.
10. Show that if L is a polytime language, then L^* is also a polytime language.

The End