Software Engineering 4I03 Michael Soltys

DAY CLASS
DURATION OF EXAMINATION: 3 Hours
MCMASTER UNIVERSITY FINAL EXAMINATION December 2004

THIS EXAMINATION PAPER INCLUDES 2 PAGES AND 10 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

No aids allowed.

All questions worth 10 marks, for a total of 100.

1. Explain precisely how to restrict the transition function of Turing machines so that the resulting model of computation recognizes exactly the set of context-free languages. Do the same for regular languages.

2. Let $M = (\{p,q,r,s\},\{0,1\},\delta,p,\{q,s\})$ be an NFA where δ is given by the following table:

δ	0	1
p	$\{q,s\}$	$\{q\}$
q	$\{r\}$	$\{q,r\}$
r	$\{s\}$	$\{p\}$
s	$\emptyset$	$\{p\}$

Construct an NFA that accepts $\overline{L(M)}$.

3. Show that if L is a regular language, then so is the following language:

$$\mathrm{SQRT}(L) = \{x \mid \exists y \text{ such that } |y| = |x|^2 \text{ and } xy \in L\ \}$$

You may assume the following result to do this question: if A, B are two regular languages, then the language defined by

$$C(A,B) = \{x \in A \mid \exists y \text{ such that } |y| = |x|^2 \text{ and } y \in B\ \}$$

is also regular.

4. Let G be the context-free grammar with rules:

$$S \longrightarrow AaSbB|\varepsilon \qquad A \longrightarrow aA|a \qquad B \longrightarrow bB|\varepsilon$$

(a) Show that G is ambiguous.

(b) Find an un-ambiguous context-free grammar equivalent to G.

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(c) For this grammar we were able to show that it is ambiguous; explain why this does not contradict the fact that $L = \{G|\ G \text{ is an ambiguous CFG }\}$ is not recursive.

5. Show that $L = \{0^n 1^m | m \leq n^2\}$ is not a context-free language.

6. Give a formal description of a Turing machine which on input $w \in \{0,1\}^*$ computes the number of 1s in w in unary. That is, on input 011010001 the Turing machine would write 1111 on the tape and halt. More precisely, $q_0 011010001 \vdash^* q_f 1111$, and in general $q_0 w \vdash^* q_f 1^i$ where i is the number of 1s in w. Explain clearly how δ works, what are the states and the different stages of the computation.

   Hint: to simplify, we allow 3 kinds of moves of the head: L (left), R (right), and $-$ (stay in place).

7. Consider the language of pairs of CFGs G_1, G_2 such that $L(G_1) \cap L(G_2) = \emptyset$. That is, $L = \{(G_1, G_2) | L(G_1) \cap L(G_2) = \emptyset\}$. The language L is a set of strings encoding such pairs; give a high level description of one such encoding (i.e., how to encode a pair of grammars with a string of 0s and 1s?). Is L decidable? Justify your answer.

8. Show that the Post Correspondence Problem (PCP) over the alphabet $\Sigma = \{a\}$ is decidable.

9. An Enumerator is a Turing machine E with two tapes: a work tape and an output tape. It takes no input, and when turned on, E will start computing and writing strings on the output tape (separated by, say, #). We say that $w \in L(E)$ iff w appears at some point on the output tape (i.e., the output tape contains $\ldots \#w\# \ldots$).

   (a) Show that a language L is Recursively Enumerable (RE) iff there exists an enumerator E such that $L = L(E)$.

   (b) Show that a language L is recursive if both L and $\overline{L}$ have enumerators.

   (c) Does every RE language have an enumerator which lists its strings in order of increasing length? (That is, if $\ldots \#w_i\# \ldots \#w_j\# \ldots$, then $|w_i| \leq |w_j|$.)

10. Let $L \in \text{TIME}(f(n))$. Show that for any $\varepsilon > 0$, $L \in \text{TIME}(\varepsilon \cdot f(n) + n + 2)$.

    Hint: increase the "word size" appropriately.

The End