Tests: Sep 24, Oct 15, Nov 5, Nov 26 in ABB-B163 (Test room)

Alphabet: Finite, non-empty set of distinct symbols, denoted Σ.

e.g. $\Sigma = \{0, 1\}$ (binary alphabet)
$\Sigma = \{a, b, c, \ldots, z\}$ (lower-case letters alphabet)

String: (also called word) finite (ordered) sequence of symbols chosen from some alphabet.

e.g. 010011101011

Empty string: ε, $|\varepsilon| = 0$, is in any Σ by default.

$|w|$ length of the string w.

e.g. $|010011101011| = 12$

Σ^k is the set of strings over Σ of length *exactly* k.

e.g. If $\Sigma = \{0, 1\}$, then $\Sigma^0 = \{\varepsilon\}$, $\Sigma^1 = \Sigma$, $\Sigma^2 = \{00, 01, 10, 11\}$, etc.

How many strings are in Σ^k?

Kleene's Star: Σ^* is the set of all strings over Σ.

$$\Sigma^* = \Sigma^0 \cup \underbrace{\Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \ldots}_{=\Sigma^+}$$

Concatenation: Let x, y be strings.

If $x = a_1 a_2 \ldots a_m$ and $y = b_1 b_2 \ldots b_n$, then

$xy = a_1 a_2 \ldots a_m b_1 b_2 \ldots b_n$

Note $w\varepsilon = \varepsilon w = w$.

Language: A collection of strings. L is a language over the alphabet Σ if $L \subseteq \Sigma^*$.

e.g., $L = \{\varepsilon, 01, 0011, 000111, \ldots\}$

Note that $\{\varepsilon\} \neq \emptyset$; one is the language consisting of the single string ε, and the other is the empty language.

Problem: Is a given string in a given language?

e.g. $L = \{\underbrace{10}_{2}, \underbrace{11}_{3}, \underbrace{101}_{5}, \underbrace{111}_{7}, \ldots\}$.

$L \subseteq \{0, 1\}^*$

$w \in L$ iff $w \in \{0, 1\}^*$ encodes a prime number in binary notation.

Deterministic Finite Automaton (DFA):

1. Finite set of states Q
2. Finite set of input symbols Σ
3. Transition function $\delta : Q \times \Sigma \longrightarrow Q$
4. Start state q_0
5. A set of final (accepting) states.

$A = (Q, \Sigma, \delta, q_0, F)$

Given $q, a,$ $\delta(q, a) = p$.

To see whether A accepts a string w, we "run" A on $w = a_1 a_2 \ldots a_n$ as follows:

$\delta(q_0, a_1) = q_1$, $\delta(q_1, a_2) = q_2$, until $\delta(q_{n-1}, a_n) = q_n$.

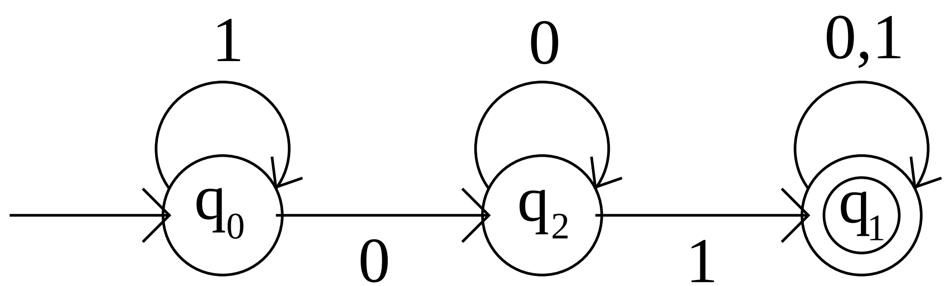
Accept iff $q_n \in F$.

Consider $L = \{w |\ w \text{ is of the form } x01y \in \Sigma^* \ \}$ where $\Sigma = \{0, 1\}$.

We want to specify a DFA $A = (Q, \Sigma, \delta, q_0, F)$ that accepts all and only the strings in L.

$\Sigma = \{0, 1\}$, $Q = \{q_0, q_1, q_2\}$, and $F = \{q_1\}$.

Transition Diagram:



Transition Table:

	0	1
q_0	q_2	q_0
q_1	q_1	q_1
q_2	q_2	q_1

Extended Transition Function: Given δ, its ETF is $\hat{\delta}$ defined inductively:

Basis Case: $\hat{\delta}(q, \varepsilon) = q$

Induction Step: if $w = xa$, $w, x \in \Sigma^*$ and $a \in \Sigma$, then

$$\hat{\delta}(q, w) = \hat{\delta}(q, xa) = \delta(\hat{\delta}(q, x), a)$$

Thus: $\hat{\delta} : Q \times \Sigma^* \longrightarrow Q$.

$w \in L(A) \iff \hat{\delta}(q_0, w) \in F$

Here $L(A)$ is the set of all those strings (and only those) which are accepted by A.

Language of a DFA: $L(A) = \{w | \hat{\delta}(q_0, w) \in F\}$

Regular Languages: L is *regular* iff there exists a DFA A such that $L = L(A)$.