Algebraic Laws for Reg Exps

$L + M = M + L$ (commutativity of $+$)

$(L + M) + N = L + (M + N)$ (associativity of $+$)

$(LM)N = L(MN)$ (associativity of concatenation)

$LM = ML$?

$\emptyset + L = L + \emptyset = L$ ($\emptyset$ identity for $+$)

$\varepsilon L = L\varepsilon = L$ (ε identity for concatenation)

$\emptyset L = L\emptyset = \emptyset$ ($\emptyset$ annihilator for concatenation)

$L(M + N) = LM + LN$ (left-distributivity)

$(M + N)L = ML + NL$ (right-distributivity)

$L + L = L$ (idempotent law for union)

Laws with closure:

$(L^*)^* = L^*$

$\emptyset^* = \varepsilon$

$\varepsilon^* = \varepsilon$

$L^+ = LL^* = L^*L$

$L^* = L^+ + \varepsilon$

Test for Reg Exp Algebraic Law:

To test whether $E = F$, where E, F are reg exp with variables $(L, M, N, \ldots)$, convert E, F to concrete reg exp C, D by replacing variables by symbols. If $L(C) = L(D)$, then $E = F$.

Ex. To show $(L + M)^* = (L^*M^*)^*$ replace L, M by a, b, to obtain $(a + b)^* = (a^*b^*)^*$.

Pumping Lemma: Let L be a regular language. Then there exists a constant n (depending on L) such that for all $w \in L$, $|w| \geq n$, we can break w into three parts $w = xyz$ such that:

1. $y \neq \varepsilon$
2. $|xy| \leq n$
3. For all $k \geq 0$, $xy^kz \in L$

Proof: Suppose L is regular. Then there exists a DFA A such that $L = L(A)$. Let n be the number of states of A. Consider any $w = a_1a_2 \ldots a_m$, $m \geq n$:

$$\underset{p_0}{\uparrow} \overbrace{a_1 \underset{p_1}{\uparrow} a_2 \underset{p_2}{\uparrow} a_3 \ldots a_i}^{x} \underset{p_i}{\uparrow} \overbrace{a_{i+1} \ldots a_j}^{y} \underset{p_j}{\uparrow} \overbrace{a_{j+1} \ldots a_m}^{z} \underset{p_m}{\uparrow}$$