Relations: R is a *relation* on two sets A, B iff $R \subseteq A \times B$.

e.g. $R = \{(m, n) |\ m - n \text{ is even }\} \subseteq \mathbb{Z} \times \mathbb{Z}$.
So $(3, 5), (2, -4) \in R$, but $(-2, 1) \notin R$.

R is an *equivalence relation* if it is

1. Reflexive: for all a, $(a, a) \in R$
2. Symmetric: for all a, b, $(a, b) \in R \Rightarrow (b, a) \in R$
3. Transitive: for all a, b, c, $(a, b) \in R$ and $(b, c) \in R$, implies that $(a, c) \in R$.

If R is an equivalence relation, and $(a, b) \in R$, then we write $a \equiv_R b$ or just $a \equiv b$.

Equivalence class: $[a] = \{x | x \equiv a\}$

Theorem: Consider an equivalence relation. Then the following hold:

1. $a \in [a]$
2. $a \equiv b \iff [a] = [b]$
3. $a \not\equiv b$ then $[a] \cap [b] = \emptyset$
4. any two equivalence classes are either equal or disjoint.

Proof: 3. prove the contra-positive: suppose $[a] \cap [b] \neq \emptyset$, so there exists an $x \in [a] \cap [b]$.

By definition, $x \equiv a$ and $x \equiv b$.

By symmetry and transitivity, $a \equiv b$.