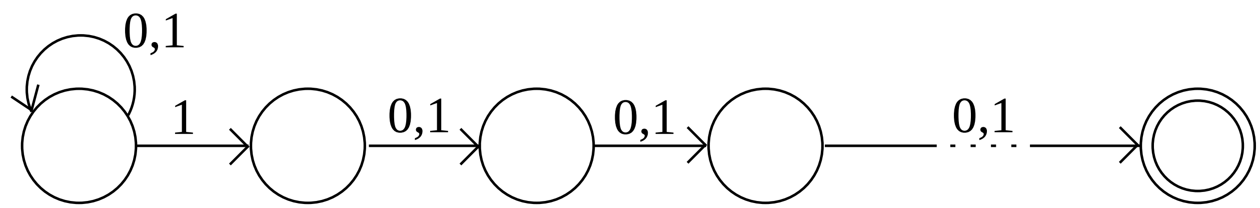
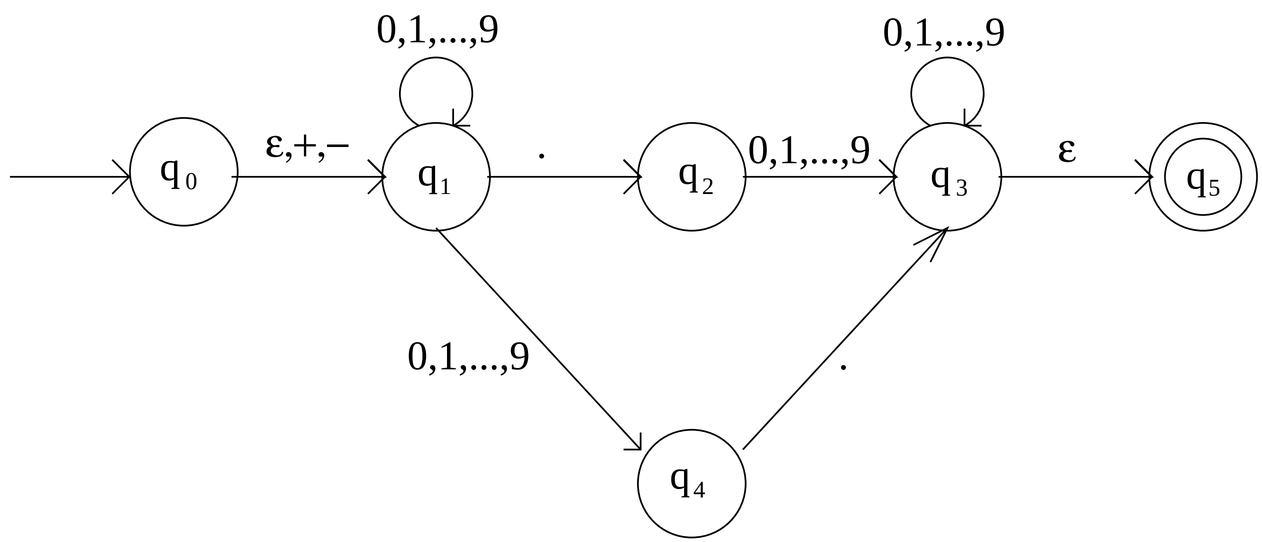
$L_n = \{w|\ n\text{-th symbol from the end is } 1\ \}$
What is an NFA for L_n



At least how many states does any DFA recognizing L_n require?

NFA with ε transitions: ε-NFA: $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \longrightarrow \mathcal{P}(Q)$



To define $\hat{\delta}$ for ε-NFAs we need the concept of ε-closure.

Given q, ε-close(q) is the set of all states p which are reachable from q by following arrows labeled by ε.

Formally, $q \in \varepsilon\text{-close}(q)$, and if $p \in \varepsilon\text{-close}(q)$, and $p \xrightarrow{\varepsilon} r$, then $r \in \varepsilon\text{-close}(q)$.

$\hat{\delta}(q, \varepsilon) = \varepsilon\text{-close}(q)$

Suppose $w = xa$, $\hat{\delta}(q, x) = \{p_1, p_2, \ldots, p_n\}$,
and $\cup_{i=1}^{n} \delta(p_i, a) = \{r_1, r_2, \ldots, r_m\}$,
then

$$\hat{\delta}(q, w) = \cup_{i=1}^{m} \varepsilon\text{-close}(r_i)$$

Theorem: DFAs and ε-NFAs are equivalent.

Proof: Slightly modified subset construction.

$q_0^D = \varepsilon\text{-close}(\{q_0^N\})$

$\delta_D(R, a) = \cup_{r \in R} \varepsilon\text{-close}(\delta_N(r, a))$

Given a set of states S, its ε-closure is the union of the ε-closures of its members.

The states of D are those subsets $S \subseteq Q_N$ which are equal to their ε-closures.

Corollary: A language is regular
$\iff$ it is recognized by some DFA
$\iff$ it is recognized by some NFA
$\iff$ it is recognized by some ε-NFA