Name ______________________ Student No. ______________________

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

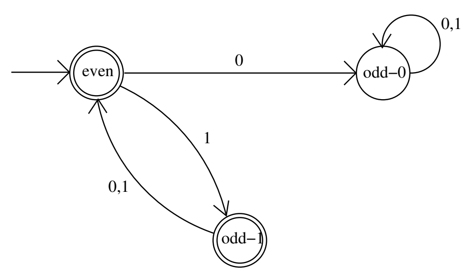
Total Marks: 50

[10] 1. Consider the following language (over the alphabet $\Sigma = \{0, 1\}$):

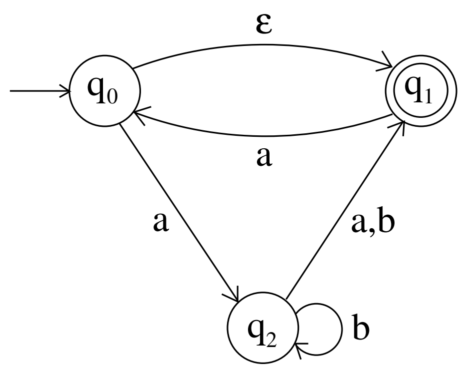
$$L = \{w| \text{ every odd position of } w \text{ is a } 1\ \}$$

Design a DFA D that recognizes L.

Solution:



[10] 2. Convert the following ε-NFA to a DFA using the subset construction:



Solution: Let $Q_D = \{\emptyset, \{q_0\}, \{q_1\}, \{q_2\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$ and let those states be denoted by $\{A, B, C, D, E, F, G, H\}$, respectively.

Using $\delta_D(S, x) = \varepsilon\text{-close}(\cup_{q \in S} \delta_N(q, x))$ we obtain:

$\delta(A, a/b) = A/A$
$\delta(B, a/b) = D/A$
$\delta(C, a/b) = E/A$
$\delta(D, a/b) = C/A$
$\delta(E, a/b) = H/A$
$\delta(F, a/b) = G/G$
$\delta(G, a/b) = E/G$
$\delta(H, a/b) = H/G$

The accepting states are C, E, G, H.

[10] 3. Consider a new kind of finite automaton called an all-path-NFA. An all-path-NFA A is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that recognizes $x \in \Sigma^*$ if *every* possible computation of A on x ends in a state from F. Note, in contrast, that an ordinary NFA accepts a string if *some* computation ends in an accept state.

Prove that all-paths-NFAs recognize the class of regular languages.

Solution: Every regular language can be recognized by an all-path-NFA since a DFA is a restricted kind of an all-path-NFA (where there is always exactly one path).

Every all-path-NFA can be converted to an equivalent DFA using the subset construction, and defining F_D as the set of all subsets of F_N except the empty one.

[10] 4. Show, using the Myhill-Nerode Theorem, that $L = \{0^m 1^n | m \neq n\} \subseteq \{0,1\}^*$ is not a regular language.

Solution: We show that L has infinitely many equivalence classes (where the equivalence relation is that of indistinguishability of strings wrt L).

Consider $[0^1], [0^2], [0^3], \ldots$. Take any $i \neq j$ $(i, j \in \mathbb{N})$. Then $[0^i] \neq [0^j]$ because $0^i 1^i \notin L$, while $0^j 1^i \in L$. Thus $z = 1^i$ distinguishes 0^i and 0^j. Hence all those equivalence classes are distinct, and there are infinitely many of them.

By the Myhill-Nerode Theorem, L is not regular.