SE4I03 Test 2 Fall 2004

Name ______________________ Student No. ______________________

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: 50

[10] 1. Write a regular expression for the language consisting of strings of 0's and 1's with *no* pair of consecutive 1's (i.e., 11 never occurs as a substring).

Solution: $(1+\varepsilon)(00^*(1+\varepsilon))^*$

[10] 2. Consider the DFA A given by:

	0	1
q_1	q_2	q_1
q_2	q_2	q_1

State q_1 is the initial state, and state q_2 is the accepting state.

(a) Give a regular expression R such that $L(A) = L(R)$.

(b) The above DFA is very simple, so it is not necessary to compute the $R_{ij}^{(k)}$'s to obtain it (we can obtain R by inspection). Suppose, however, that we want to compute the $R_{ij}^{(k)}$'s; explain briefly how to compute $R_{ij}^{(0)}$ and how to compute $R_{ij}^{(k)}$ for $k > 0$, and once you computed them all, how to obtain R.

Solution: $R = (0+1)^*00^*$. See pages 92 and 93 in the textbook for part (b).

[10] 3. Prove, using the Pumping Lemma, that the following two languages are *not* regular:

(a) $\{0^n 1^m | \ n \leq m\ \}$

(b) $\{0^{2^n} | \ n \geq 0\ \}$

Solution: For (a), let p be the pumping length, and choose $0^p 1^p$, which is in the language, then pump y twice, to obtain a prefix of 0's longer than the suffix of 1's; this gives a contradiction since the resulting string cannot be in the language, while the PL says that it is.

For (b) let p be the pumping length, and consider any k such that $2^k > p$. By the PL, $0^{2^k} = xyz$, and xy^iz is in the language for any i. So, $0^{2^k+|y|}$ is in the language, where $0 < |y| \leq p$. Now simply choose k big enough so the $2^k < 2^k + |y| < 2^{k+1}$, and we have our contradiction.

[10] 4. Consider the following two DFAs:

	0	1
A	A	B
B	A	B

Initial state is A, and the accepting state is also A.

	0	1
C	D	E
D	D	E
E	C	E

Initial state is C and the accepting states are C and D.

Using the Table-Filling algorithm, establish whether these two DFAs are equivalent.

Solution: They are equivalent; see the table on page 158 in the textbook.

[10] 5. Give a procedure that tests whether a DFA A accepts exactly 7 strings. (You do not have to prove that your procedure is correct; just state what the procedure is. Your procedure must produce the correct answer and *always* halt. Hint: think of the procedure for testing whether $L(A)$ is infinite.)

Solution: Given A, we calculate its number of states; say it is n. We now test to see if there is a string w of length $n \leq |w| \leq 2n$ accepted by A. If the answer if "yes", then we know that A accepts infinitely many strings, and therefore we answer "no" to the question whether it accepts exactly 7 strings. If the answer is "no" (i.e., no string w of length $n \leq |w| \leq 2n$ is accepted by A), then we know that if A accepts any string, it has to be of length $< n$. There are finitely many such strings. We test them all, and see how many are accepted by A; if 7, we answer "yes", otherwise we answer "no".