Due on Tuesday May 22 in class

1. Consider the language (over $\{0, 1, \texttt{\#}\}$)

$$L = \{\texttt{\#}b_k(0)\texttt{\#}b_k(1)\texttt{\#}b_k(2)\texttt{\#}\ldots\texttt{\#}b_k(2^k-1)\texttt{\#} | k \geq 0\}$$

   where $b_k(i)$ is the k-bit binary representation of $i \leq 2^k - 1$. Show that this language is (i) not regular, and (ii) decidable in space $O(\log\log n)$.

2. Show that the languages which can be decided by Turing machines which have a read-only input tape, and a work-tape where space is bounded by a function in $o(\log\log n)$ (little-oh: $g(n) \in o(f(n))$ if $\lim_{n\to\infty} g(n)/f(n) = 0$) are regular.

   Hint: First show that if a language is decided by such a two-tape Turing machine where the work-tape space is actually bounded by a constant, then the language is regular. Then show that if a language is not regular, then the work-tape requires $\Omega(\log\log n)$ space (where this is the "weaker" version of Ω saying that for some constante c, $c\log\log n$ is a lower bound for *infinitely many* n's). For the second part you may want to use crossing sequences.

3. Say that a language is *stingy* if there is a polynomial $p(n)$ such that the number of strings of a given length n in the language is at most $p(n)$. Formally, L is stingy if there exists a polynomial $p(n)$ such that $|\{x \in S : |x| = n\}| \leq p(n)$.

   Show that if there is a stingy language L which is hard for **co-NP** wrt[1] log-space many-one reductions (i.e., $\leq^m_{\mathbf{L}}$), then in fact $\mathbf{P} = \mathbf{NP}$.

[1] It means "with respect to".