Filling out details in the proof of Claim 8 in

Feasible Combinatorial Matrix Theory by Fernandez & Soltys

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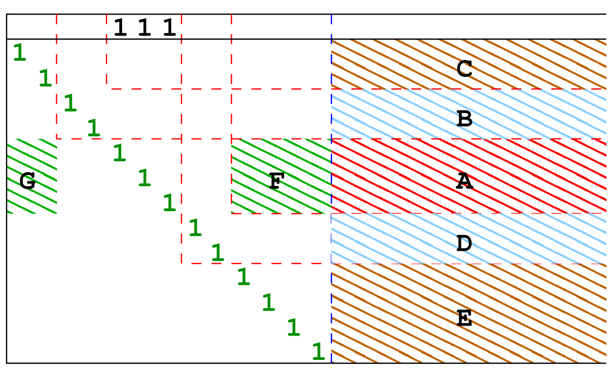
In this note we provide details for the proof of Claim 8 in the paper *Feasible combinatorial matrix theory* by Ariel German Fernandez and Michael Soltys, which is going to be presented at MFCS'13. The proof contained therein is correct, but, due to the conference proceedings space limitations, incomplete. In particular, in this note we expand on the last paragraph in that proof[1]. That paragraph is a very "bare-bones" outline of the proof of the inductive step — part of the inductive proof employed to prove Claim 8.

The induction is on the number of rows, where A_k contains the first k rows of A (this is not crucial, but we count the rows from the bottom). We denote the first row of A (in the usual sense, the top row), as row $k+1$. If row $k+1$ is *not* covered by $C_{\mathcal{S}}$ (which consists of the vertical lines going through the points in $\mathcal{S}$), then we pick a 1 from row $k+1$ that is not covered and add it to our selection. This is the easy case; we now assume that $C_{\mathcal{S}}$ does cover row $k+1$.

We re-arrange A_{k+1} by permuting its rows and columns so that it is of the following form:

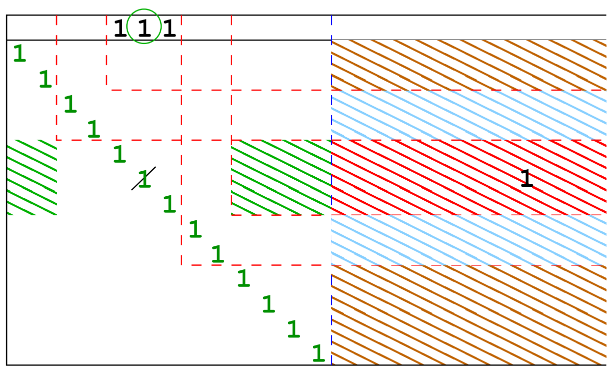


The re-arrangement of A_{k+1}, giving it the above form, is obtained as follows: first, we place the selected 1s (the green 1s) from A_k along the main diagonal of A_k. This permutation might rearrange the columns of row $k+1$, but row $k+1$ continues being in the same position — the first row from the top. We

[1]See the last paragraph on page 5 of the final submission — `http://bit.ly/1B6k3h9`

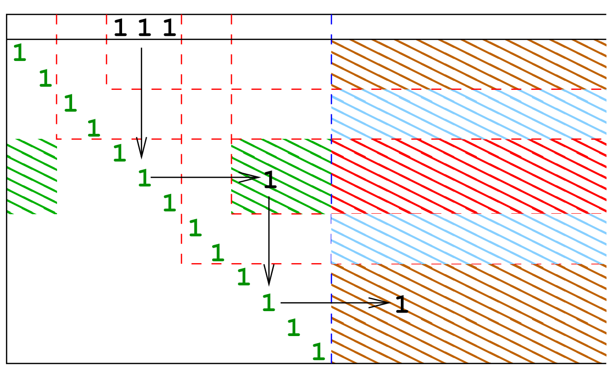
then re-arrange the green 1s so that areas B and D contain only zeros (note that these areas might be empty).

Suppose first that area A contains a 1; say that this 1 is in row i of the matrix. Then pick the green 1 along row i, remove it from the selection, and instead add to the selection the corresponding black 1 (i.e., the 1 in row $k+1$ that is in column $i-1$), and the 1 from area A. The diagram below represents the situation: the black 1 in row $k+1$ with the green circle around it is the selection from row $k+1$. The crossed-out green 1 has be de-selected, and replaced by the black 1 in area A.



Suppose that area A is all zeros. Suppose that areas G and F are also all zeros; then we can cover A_{k+1} with k lines as follows: horizontal lines covering C,B,D,E and vertical lines covering the green 1s between G and F. But this contradicts the assumption that A_{k+1} requires $k+1$ lines to be covered.

Thus, under the assumption that area A is all zeros, we know that there must be 1s in area G or area F. As G and F are symmetric cases, assume that the 1 is in F. Then we re-arrange the selection as in the following diagram.



Following the direction of the path, the black 1 from row $k+1$ is selected; the green 1 is deleted and replaced by the black 1 in area F; the second green 1 on the path is deleted, and replaced by the black 1 in area E. Also note that if E were empty, so would be F.