CSU Channel Islands
MATH 354 Midterm
October 25, 2017

Duration: 1 hour and 10 minutes

No Aids Allowed.
There are 4 questions worth a total of 20 marks (5 marks each).
Answer all questions on the question paper.
Use backs of pages.

Please complete this section:

Name (please print): ____________________

For use in marking:

1.__________ /5

2.__________ /5

3.__________ /5

4.__________ /5

Total: __________ /20

1. Recall the Gale-Shapley algorithm:

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1: Stage 1: b_1 chooses his top g and M_1 ⟵ {(b_1, g)}
2: for s = 1, ..., s = |B| − 1, Stage s + 1: do
3:     M ⟵ M_s
4:     b* ⟵ b_{s+1}
5:     for b* proposes to all g's in order of preference: do
6:         if g was not engaged: then
7:             M_{s+1} ⟵ M ∪ {(b*, g)}
8:             end current stage
9:         else if g was engaged to b but g prefers b*: then
10:            M ⟵ (M − {(b, g)}) ∪ {(b*, g)}
11:            b* ⟵ b
12:            repeat from line 5
13:        end if
14:    end for
15: end for
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Assuming that $|B| = |G| = n$, show that at the end of stage n, M_n will be a stable marrige. Recall that a marriage is stable if there is no blocking pair (a pair (b, g) from $B \times G$ such that b and g are not partners in M but b prefers g to $p_M(b)$ and g prefers b to $p_M(g)$).

2. Let MaxCST be a variation of the MCST (Minimum Cost Spanning Tree) where we are interested in a Maximum Cost Spanning Tree.

   Does the following modification of Kruskal's algorithm (edge weights given in non-increaseing rather than non-decreasing order) always produce a MaxCST?

   Sort the edges: $c(e_1) \geq c(e_2) \geq \ldots \geq c(e_m)$
   $T \longleftarrow \emptyset$
   for $i : 1..m$ **do**
       if $T \cup \{e_i\}$ has no cycle **then**
           $T \longleftarrow T \cup \{e_i\}$
       end if
   end for

   Justify your answer.

3. Explain why classical bit multiplication of two n-bit integers takes $O(n^2)$, but the Divid and Conquer approach only $O(n^{1.59})$ operations.

   Hint: use the fact that n bit integers can be broken up into two $n/2$-bit integers, so $z = z_1 \cdot 2^{n/2} + z_0$ where z has n-bits, while z_1, z_0 have $n/2$ bits. (Ignore issues of divisibility; $n/2$ is really $\lfloor n/2 \rfloor$ but overlook it for the sake of simplicity.)

4. Consider the following variant of the Longest Monotone Subsequence problem. The input is $d, a_1, a_2, \ldots, a_d \in \mathbb{N}$, but the output is the length of the longest subsequence of $a_1, a_2, \ldots, a_d$, where any two consecutive members of the subsequence differ by at most 1. For example, the longest such subsequence of $\{7, 6, 1, 4, 7, 8, 20\}$ is $\{7, 6, 7, 8\}$, so in this case the answer would be 4. Give a dynamic programming solution.