

Name _____

No aids allowed. Answer all questions on test paper. Use backs of sheets if necessary.

Claim: We get the same “boy-optimal” matching from the Gale-Shapley algorithm regardless of the ordering of the boys.

A pair (b, g) is *feasible* if there exists a stable matching with (b, g) . We say that g is b 's optimal partner if g is as good as b can get among b 's feasible pairings. We say that a matching is *boy-optimal* if every boy is paired with his highest ranked feasible partner.

Follow these steps to show that GS produces a boy-optimal matching (arguing by contradiction), and conclude the Claim.

1. Let b be the first boy rejected by his optimal (ranked nr. 1) g . Explain why this means that g has already been paired with some b' and g prefers b' to b (i.e., $b' <_g b$).
2. Explain why g is at least as desirable to b' as his own optimal partner.
3. Conclude GS produces a boy-optimal matching, and conclude that GS produces the same matching regardless of the ordering of the b s.