**CSU Channel Islands**
**MATH 354 Exam Part 1**
**December 4, 2017**

Duration: 1 hour and 10 minutes

No Aids Allowed.
There are 3 questions worth a total of 15 marks (5 marks each).
Answer all questions on the question paper.
Use backs of pages.

**Please complete this section:**

Name (please print): ______________________________

**For use in marking:**

1.__________ /5

2.__________ /5

3.__________ /5

Total: __________ /15

1. Recall the Gale-Shapley algorithm:

1: Stage 1: $b_1$ chooses his top $g$ and $M_1 \longleftarrow \{(b_1, g)\}$<br>
2: **for** $s = 1, \ldots, s = |B| - 1$, Stage $s + 1$: **do**<br>
3:     $M \longleftarrow M_s$<br>
4:     $b^* \longleftarrow b_{s+1}$<br>
5:     **for** $b^*$ proposes to all $g$'s in order of preference: **do**<br>
6:         **if** $g$ was not engaged: **then**<br>
7:             $M_{s+1} \longleftarrow M \cup \{(b^*, g)\}$<br>
8:             end current stage<br>
9:         **else if** $g$ was engaged to $b$ but $g$ prefers $b^*$: **then**<br>
10:             $M \longleftarrow (M - \{(b, g)\}) \cup \{(b^*, g)\}$<br>
11:             $b^* \longleftarrow b$<br>
12:             repeat from line 5<br>
13:         **end if**<br>
14:     **end for**<br>
15: **end for**

(a) What does it mean for a pair $(b, g)$ to be *feasible*?

(b) What does it mean for a matching to be *boy-optimal*?

(c) Show that the algorithm, as given above, produces a boy-optimal matching.

2. Consider the MCST (Minimum Cost Spanning Tree) problem. Suppose that edge $e_1$ in $G$ has a smaller cost than all the other edges (i.e., $c(e_1) < c(e_i)$ for all $i > 1$). Show that ***every*** MCST for $G$ must contain $e_1$.

   For your reference, here is Kruskal's algorithm.

   Sort the edges: $c(e_1) \leq c(e_2) \leq \ldots \leq c(e_m)$
   $T \longleftarrow \emptyset$
   **for** $i : 1..m$ **do**
       **if** $T \cup \{e_i\}$ has no cycle **then**
           $T \longleftarrow T \cup \{e_i\}$
       **end if**
   **end for**

3. Explain why classical bit multiplication of two $n$-bit integers takes $O(n^2)$, but the Divid and Conquer approach only $O(n^{1.59})$ operations.

   Hint: use the fact that $n$ bit integers can be broken up into two $n/2$-bit integers, so $z = z_1 \cdot 2^{n/2} + z_0$ where $z$ has $n$-bits, while $z_1, z_0$ have $n/2$ bits. (Ignore issues of divisibility; $n/2$ is really $\lfloor n/2 \rfloor$ but overlook it for the sake of simplicity.)