Please submit one assignment per group; form the groups at the beginning of the course, and work together on all assignments (except the final exam which will be submitted individually).

Prove the following claims about the G-S Algorithm (Algorithm 1.7). This sequence of claims builds toward a proof of correctness of the G-S Algorithm.

1. From the moment that g receives her first proposal, g remains engaged. Also, her sequence of partners gets better and better (in terms of her list of preferences).
2. The sequence of g's to whom a particular b proposes gets worst and worst (again, in terms of his list of preferences).
3. The following is an invariant of the G-S Algorithm: *if b is free (not engaged) at some point in the execution of the algorithm, then there is a g to whom he has not yet proposed.*
4. The set of pairs M at the end of the execution of the algorithm constitutes a *perfect matching.* (Start by defining what a "perfect matching" is.)
5. The set of pairs M at the end of the execution of the algorithm constitutes a *stable matching.* (Start by defining what a "stable matching" is.)
6. Give an example of a B, G with corresponding lists of preferences for which there is more than one stable matching.
7. Recall the definition of a *feasible pair* in the textbook (pg. 17). Let's say that g is the *best feasible pair* for b, if (b, g) is a feasible pair, and there is no g' such that:

   $g' <_b g$ **and** (b, g') is also a feasible pair.

   For any given b, let $\mathcal{B}(b)$ be b's best feasible pair. Finally, let $M^* = \{(b, \mathcal{B}(b)) : b \in B\}$. Show that the G-S Algorithm yields M^*.
8. Show that any re-ordering of B still yields M^*, that is, the G-S Algorithm is independent of the order of the boys.
9. Show that in M^*, each g is paired with her worst feasible partner.
10. Assess the running time (complexity) of the algorithm in terms of Big-Oh complexity.