

Name \_\_\_\_\_

*No aids allowed. Answer all questions on test paper. Use backs of sheets if necessary.*

The question below is Problem 1.4 in the textbook.

Suppose that the precondition in the algorithm is changed to say: “ $x \geq 0 \wedge y > 0 \wedge x, y \in \mathbb{Z}$ ,” where  $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ . Is the algorithm still correct in this case? What if it is changed to the following: “ $y > 0 \wedge x, y \in \mathbb{Z}$ ”? How would you modify the algorithm to work with negative values?

**Require:**  $x \geq 0 \wedge y > 0 \wedge x, y \in \mathbb{N}$

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1:  $q \leftarrow 0$ 
2:  $r \leftarrow x$ 
3: while  $y \leq r$  do
4:    $r \leftarrow r - y$ 
5:    $q \leftarrow q + 1$ 
6: end while
7: return  $q, r$ 
```

**Ensure:**  $x = (q \cdot y) + r \wedge 0 \leq r < y$