CSU Channel Islands
COMP/MATH 354 Midterm
October 24, 2018

Duration: 1 hour and 10 minutes

No Aids Allowed.
There are 4 questions worth a total of 20 marks (5 marks each).
Answer all questions on the question paper.
Use backs of pages.

Please complete this section:

Name (please print): ______________________________

For use in marking:

1.__________ /5

2.__________ /5

3.__________ /5

4.__________ /5

Total: __________ /20

1. Prove that the division algorithm terminates.

    $q \leftarrow 0$
    $r \leftarrow x$
    while $y \leq r$ **do**
        $r \leftarrow r - y$
        $q \leftarrow q + 1$
    end while
    return q, r

2. Recall the Gale-Shapley algorithm:

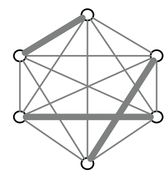
1: Stage 1: b_1 chooses his top g and $M_1 \longleftarrow \{(b_1, g)\}$
2: **for** $s = 1, \ldots, s = |B| - 1$, Stage $s + 1$: **do**
3: $\quad M \longleftarrow M_s$
4: $\quad b^* \longleftarrow b_{s+1}$
5: $\quad$ **for** b^* proposes to all g's in order of preference: **do**
6: $\quad\quad$ **if** g was not engaged: **then**
7: $\quad\quad\quad M_{s+1} \longleftarrow M \cup \{(b^*, g)\}$
8: $\quad\quad\quad$ end current stage
9: $\quad\quad$ **else if** g was engaged to b but g prefers b^*: **then**
10: $\quad\quad\quad M \longleftarrow (M - \{(b, g)\}) \cup \{(b^*, g)\}$
11: $\quad\quad\quad b^* \longleftarrow b$
12: $\quad\quad\quad$ repeat from line 5
13: $\quad\quad$ **end if**
14: $\quad$ **end for**
15: $\quad M_{s+1} \longleftarrow M$
16: **end for**
17: **return** $M_{|B|}$

(a) Show that the partners of the boys get progressively worse, and the partners of the girls get progressively better.

(b) Show that after stage s, all the boys in $\{b_1, b_2, b_3, \ldots, b_s\}$ are engaged; can the same be said of the girls in $\{g_1, g_2, g_3, \ldots, g_s\}$?

(c) What does it mean that the final marriage is boy-optimal but girl-pessimal?

3. We say that a graph is a *clique* if every node is connected to every other node. The following graph on 6 vertices is an example of a clique:



Consider Kruskal's algorithm where the input is a clique such that all edges have the same cost.

$T \longleftarrow \emptyset$
for $i : 1..m$ **do**
    if $T \cup \{e_i\}$ has no cycle **then**
        $T \longleftarrow T \cup \{e_i\}$
    end if
end for

Suppose that we pick $\lfloor n/2 \rfloor$ edges where no two of those edges share a node. The $\lfloor 6/2 \rfloor = 3$ thick edges in the graph above have this property.

Show that it is possible to order the edges in such a way that, when presented in that order to Kruskal's algorithm, they will output your selection of $\lfloor n/2 \rfloor$ as part of the tree. That is, all your edges will be in the final T (plus other edges).

4. Explain why classical bit multiplication of two n-bit integers takes $O(n^2)$, but the Divide and Conquer approach only $O(n^{1.59})$ operations.

   Hint: use the fact that n bit integers can be broken up into two $n/2$-bit integers, so $z = z_1 \cdot 2^{n/2} + z_0$ where z has n-bits, while z_1, z_0 have $n/2$ bits. (Ignore issues of divisibility; $n/2$ is really $\lfloor n/2 \rfloor$ but overlook it for the sake of simplicity.)