CSU Channel Islands
COMP/MATH 354 Midterm
March 14, 2019

Duration: 1 hour and 10 minutes

No Aids Allowed.
There are 3 questions worth a total of 15 marks (5 marks each).
Answer all questions on the question paper.
Use backs of pages.

Please complete this section:

Name (please print): _____

For use in marking:

1._____ /5

2._____ /5

3._____ /5

Total: _____ /15

1. Prove that Euclid's algorithm terminates:

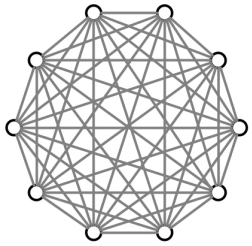
   Pre-condition: $a > 0 \wedge b > 0 \wedge a, b \in \mathbb{Z}$

   1: $m \leftarrow a$; $n \leftarrow b$; $r \leftarrow \text{rem}(m, n)$
   2: **while** $(r > 0)$ **do**
   3: $\quad m \leftarrow n$; $n \leftarrow r$; $r \leftarrow \text{rem}(m, n)$
   4: **end while**
   5: **return** n

   Post-condition: $n = \gcd(a, b)$

2. What is the PageRank of the pages in the following two (separate) networks, ignoring the *damping factor*.

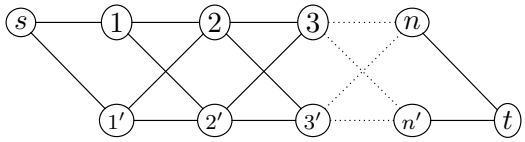
   Once you establish the PageRank for the particular number of nodes (10 in each case), establish it in general for N number of nodes for the same type of graphs.



Every page points to every other page

Every page pointing to just one (except the last one)

3. Consider the graph below:



(a) Suppose that each edge has cost exactly 1. How many Minimum Cost Spanning Trees does this graph have (as a function of n)?

(b) Now suppose that the cost of each edge is computed as follows: if it is a top layer edge or a bottom layer edge, the cost is the minimal number of nodes needed to traverse starting at s to get to it. But for edges across layers there is a tax of n.

For example, the horizontal edge leaving s ($(s, 1)$) has cost 1, but the layer-crossing edge ($(s, 1')$) has cost $n + 1$.

Which MSCS does Kruskal's algorithm output in this case?

```
1: T ⟵ ∅
2: for i : 1..m do
3:    if T ∪ {e_i} has no cycle then
4:       T ⟵ T ∪ {e_i}
5:    end if
6: end for
```