CSU Channel Islands
COMP/MATH 354 Exam
December 10, 2018
10:30–12:30 in Sierra 1422

Duration: 2 hours

No Aids Allowed.
There are 3 questions worth a total of 15 marks (5 marks each).
Answer all questions on the question paper.
Use backs of pages.

Please complete this section:

Name (please print): ____________________

For use in marking:

1.__________ /5

2.__________ /5

3.__________ /5

Total: __________ /15

1. A delivery hub coordinates the distribution of orders by packaging items in boxes, one item per box. The facility operates during business hours, say 9am to 5pm, and receives a large consignment of items to be be packaged before 9am. Each item has associated with it two parameters: its value, and a deadline which always takes place on the hour (so the possible deadlines are 10,11,12,1,2,3,4 and 5).

   For example, and item i has two parameters associated with it, a value in dollars, say \$120.45, and a deadline of 3pm. This means that i must be placed in a box by 3pm; if it cannot be placed in a box by 3pm, it will not be boxed and shipped at all.

   The delivery hub is able to box one item every 20 seconds, so there are 3 boxing slots every minute, 180 boxing slots every hour, and $8 \times 180 = 1,440$ slots during the day (9am to 5pm). There may be more items than slots on any given business day.

   Management has asked you to optimize the process, so that the largest total value of items is shipped by the end of the day, and all shipped items are shipped by their deadlines.

   Your task: Present an algorithm that manages which item is boxed in what slot during the day. Explain why (no proof required) your solution works. (Use this page, and the next for this question.)

Solution to question 1 continued . . .

2. What is the running time of the Gale-Shapley algorithm? Justify your answer, and present your estimate in Big-O notation.

1: Stage 1: b_1 chooses his top g and $M_1 \longleftarrow \{(b_1, g)\}$
2: **for** $s = 1, \ldots, s = |B| - 1$, Stage $s + 1$: **do**
3:     $M \longleftarrow M_s$
4:     $b^* \longleftarrow b_{s+1}$
5:     **for** b^* proposes to all g's in order of preference: **do**
6:         **if** g was not engaged: **then**
7:             $M_{s+1} \longleftarrow M \cup \{(b^*, g)\}$
8:             end current stage
9:         **else if** g was engaged to b but g prefers b^*: **then**
10:             $M \longleftarrow (M - \{(b, g)\}) \cup \{(b^*, g)\}$
11:             $b^* \longleftarrow b$
12:             repeat from line 5
13:         **end if**
14:     **end for**
15:     $M_{s+1} \longleftarrow M$
16: **end for**
17: **return** $M_{|B|}$

3. Design an algorithm which on input string s, for example $s = 0110110$, checks whether the string is a ***binary palindrome***, and outputs 'Yes' or 'No'. The example string just given is a palindrome, meaning that it reads the same left-to-right as right-to-left, but $s = 011$ is *not* a palindrome. Here are the steps of your solution.

   (a) Present the algorithm in pseudo-code

   (b) Propose a pre-condition and a post-condition

   (c) Propose a loop invariant

   (d) Prove by induction that the loop invariant holds

   (e) Conclude that your algorithm is correct

(Use this page and the next.)

Solution to question 3 continued ...